Anharmonic Confinement Induced Resonances: Theory vs Experiment

Peter D. Drummond, Shi-Guo Peng*, Hui Hu, Xia-Ji Liu

CAOUS Centre, Swinburne University of Technology
*Tsinghua University, Beijing

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A Detective Story in Six Parts

Harmonic Confinement Induced Resonance (HCIR)

Low-dimensional transverse resonance phenomena

Predicted by Olshanii - Phys. Rev. Lett. **81**, 938 (1998)!

- Harmonic resonance: SINGLE internal excited state
- Like a Feshbach resonance for waveguides
- Allows quantum engineering by changing trap frequency

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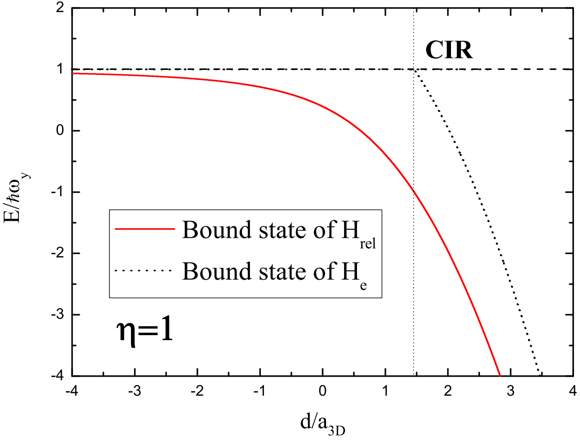
HCIR in ANISOTROPIC waveguide

Resonances at $\varepsilon = 0$, satisfy an integral equation:

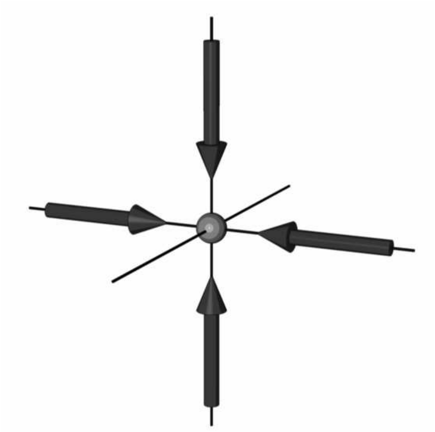
$$\mathscr{F}_e(\varepsilon, 0) = \int_0^\infty dt \left\{ \frac{exp\left(\frac{t\varepsilon}{2}\right)\sqrt{\eta}}{\sqrt{t}} \times \left[\frac{1}{\sqrt{(1-e^{-\eta t})(1-e^{-t})}} - 1 \right] - \frac{1}{t^{3/2}} \right\} = -\frac{\sqrt{\pi} d}{a_{3D}}.$$

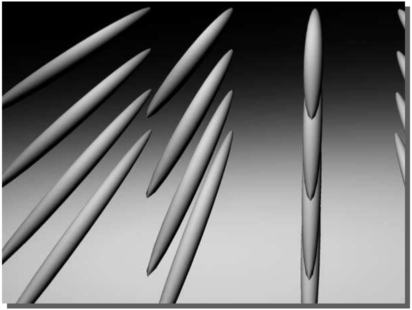
- $d = \sqrt{2\hbar/m\omega_y}$, $\eta = \omega_x/\omega_y$,
- a_{3D} = 3D scattering length
- Peng, et al, Phys. Rev. A **82**, 063633 (2010)

Physics: SINGLE resonance predicted in waveguides

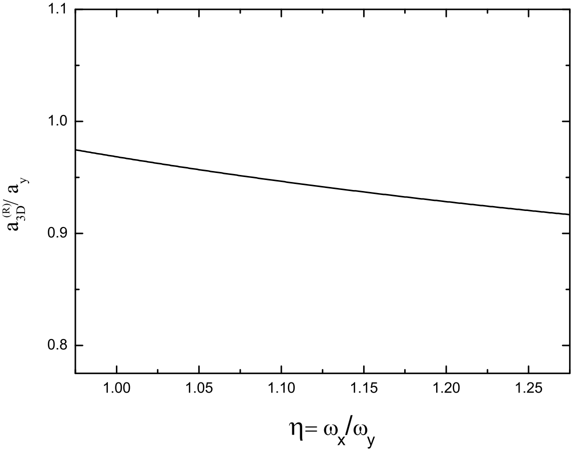


Innsbruck Cs Experiment: PRL 104, 153203 (2010)

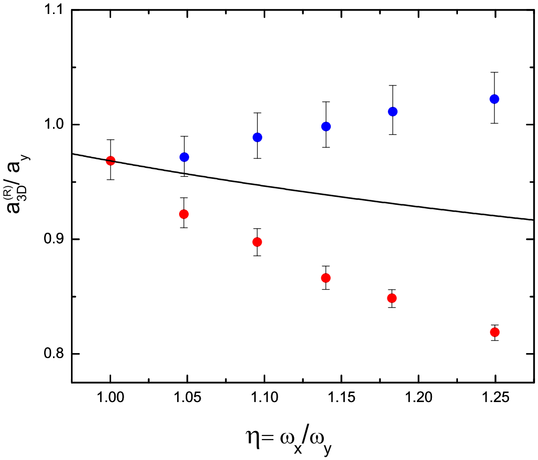




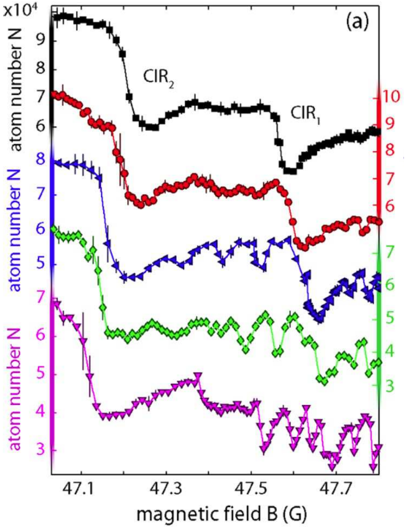
Expected HCIR resonance vs anisotropy



Innsbruck CIR Measurements *don't* fit theory!



Large anisotropy: multiple resonances - also don't fit theory!



Two-dimensional HCIR resonance only for ATTRACTIVE case

- Predicted by Petrov et al, PRL 84, 2551 (2000).

Experimental 2D resonance

- Observed resonance at Innsbruck for REPULSIVE case !!
- Similar observation with fermions by Vale group, SUT
- Resonance observed for ATTRACTIVE case at Cambridge!
- See: Frohlich et. al, PRL 106, 105301 (2011).

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Mystery - what are these new resonances??

ACIR - anharmonic waveguide resonances

NEW COLD-ATOM QUANTUM TECHNOLOGIES!

- Anharmonic confinement induced resonances - ACIR:
  - Even order multiple resonances:
  - Caused by QUARTIC nonlinearity
  - Odd order multiple resonances:
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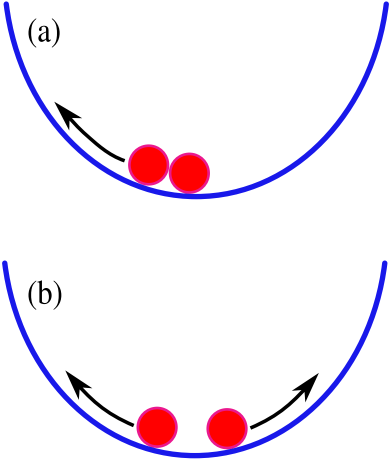
Two types of confinement resonances



(a) ACIR: MANY resonances

- Sala et. al, arXiv:1104.1561; Peng et. al., arXiv:1107.2725
- due to COM resonance of atom pairs

(b) HCIR: SINGLE resonance

- Olshanii, PRL **81**, 938 (1998)
- due to internal excitation of relative motion

Anharmonic confinement potential

For dipole trapping experiments with optical lattices, the trapping potential near a potential minimum at $\mathbf{r} = 0$ is of form:

$$\begin{aligned} U^{ext}(\mathbf{r}) &= V_x(\mathbf{r}) \sin^2\left(\frac{2\pi x}{\lambda_x}\right) + V_y(\mathbf{r}) \sin^2\left(\frac{2\pi y}{\lambda_y}\right) \\ &\approx \frac{1}{2} m \left[\omega_x^2 x^2 \left(1 + \frac{\alpha_x x^2}{d_x^2} + \frac{\mathbf{r} \cdot \nabla V_x}{V_x^0} + \ldots\right) \right. \\ &\quad \left. + \omega_y^2 y^2 \left(1 + \frac{\alpha_y y^2}{d_y^2} + \frac{\mathbf{r} \cdot \nabla V_y}{V_y^0} + \right)\right] \end{aligned}$$

Typical anharmonic parameters

Anharmonic quartic parameter:

$$\alpha_{x,y} = -\frac{1}{3}\left(2\pi d_{x,y}/\lambda\right)^2 = -8\pi^2\hbar/\left(3\lambda^2 m\omega_{x,y}\right)$$

Experiment	^{133}Cs	^{40}K
Trap frequency ω	$2\pi \times 14.5\, kHz$	$2\pi \times 80\, kHz$
Wavelength λ	$1.064 \times 10^{-6} m$	$1.064 \times 10^{-6} m$
Atomic mass m	$2.22 \times 10^{-25} kg$	$0.6635 \times 10^{-25} kg$
Length $d_{x,y}$	$0.102 \times 10^{-6} m$	$0.08 \times 10^{-6} m$
Anharmonicity $\alpha_{x,y}$	$\mathbf{-0.121}$	$\mathbf{-0.075}$

Theory with tightly bound molecules

Anharmonic energy in units of $\hbar\omega_y$

- Two non-interacting atoms in a waveguide, $\eta = \omega_x/\omega_y$:

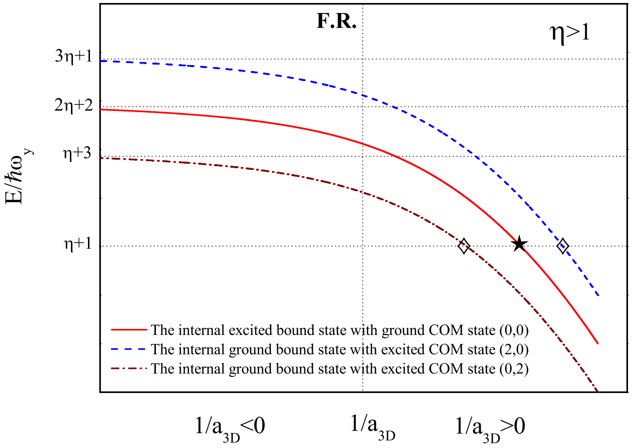
$$\varepsilon_a = \eta\,(1 + 3\alpha_x/8) + (x \leftrightarrow y)$$

- Bound Feshbach molecule: $\varepsilon_b = -\hbar/\left(m a_{3D}^2 \omega_y\right)$
- Feshbach molecule in a waveguide:

$$\varepsilon_m = \varepsilon_b + \eta\left\{N_x + \frac{1}{2} + \frac{3\alpha_x}{16}\left[N_x\,(N_x + 1) + \frac{1}{2}\right]\right\} + x \leftrightarrow y$$

- ACIR resonance: $\varepsilon_m = \varepsilon_a$

ACIR vs HCIR Resonance



General theory of ACIR resonance

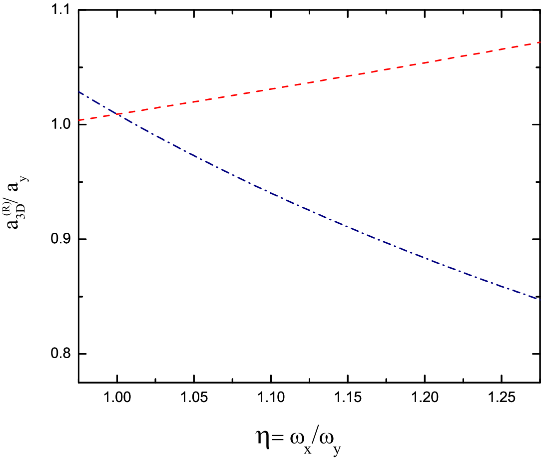
- Integral equation for bound state energy ε_b:

$$-d = \frac{a_{3D}}{\sqrt{\pi}} \int_0^\infty dt \left[\frac{\sqrt{\eta} \exp(\varepsilon_b t/2)}{\sqrt{t(1-e^{-\eta t})(1-e^{-t})}} - \frac{1}{t^{3/2}} \right].$$

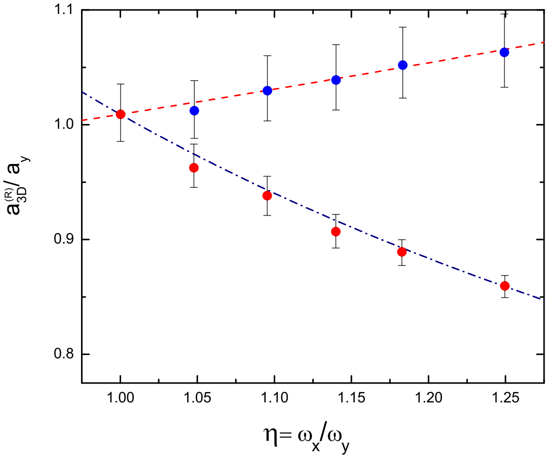
- Anharmonic ACIR resonance (N_x, N_y) when:

$$\varepsilon_b + 2\eta N_x + N_y + \alpha \left\{ \frac{3(N_y(1+N_y) + N_x(1+N_x)) - 9}{16} - \frac{2N_y + 1 + \eta(2N_x+1)}{32\varepsilon} + \frac{3(\eta^2+1)}{320\varepsilon^2} \right\} = 0$$

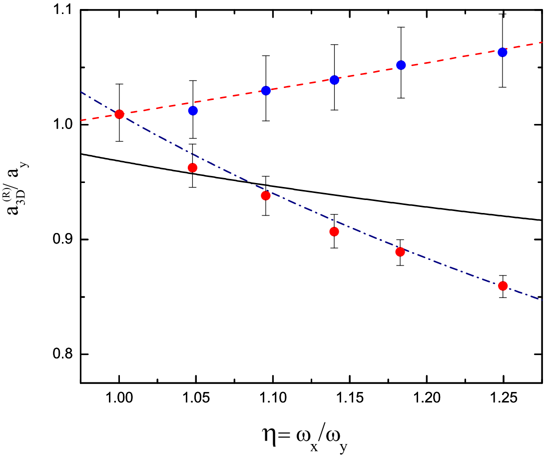
Predicted ACIR resonance vs anisotropy



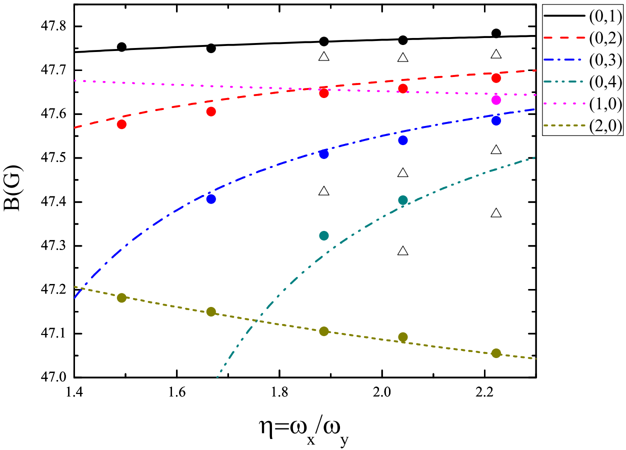
Comparison of ACIR to experiment



ACIR vs HCIR vs Experiment



Large anisotropy: theory vs. experiment



Multiple resonance summary

Multiple resonances are an ACIR signature!

- Identified 22 ACIR resonances at large anisotropy.

Also 8 unidentified resonances:

- Internal resonance with anharmonic shifts?
- Four atom mixing $\rightarrow$ pairs of molecules?

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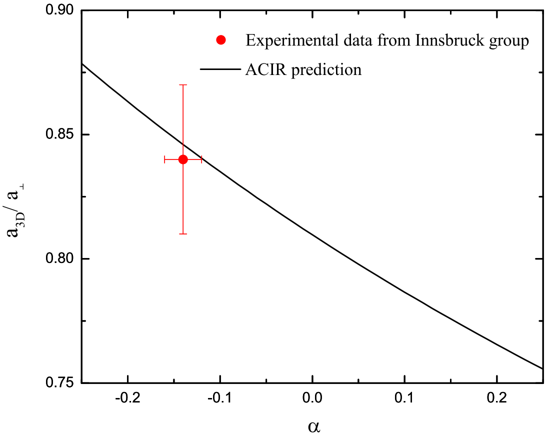
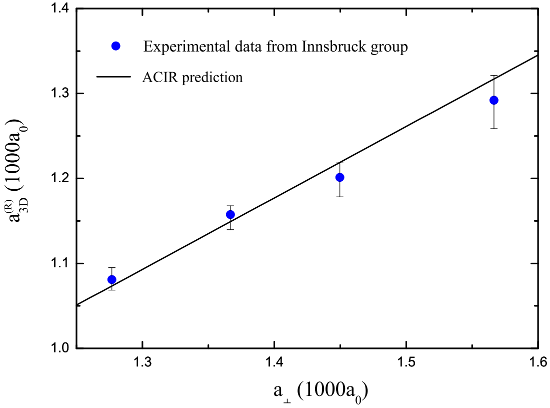
Two-dimensional limit: a_{3D} vs anharmonicity



Two-dimensional limit: a_{3D} vs confinement $a_\perp$



New type of low-dimensional nonlinear optics

Experiment in excellent agreement with ACIR theory!

- Anharmonic confinement induced COM resonances - ACIR:
  - Explains 35 newly observed resonances
  - Caused by QUARTIC and CUBIC anharmonic couplings
- Harmonic confinement induced internal resonances - HCIR:
  - Does not match any Innsbruck data! Molecular loss insensitive to HCIR?
- Unexplained resonances -
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  - Higher order 4-body anharmonic resonance?

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