

# Bose-Bose mixtures in confined dimensions



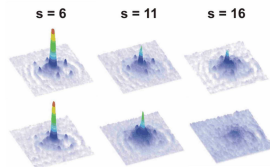
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**22nd International Conference  
on Atomic Physics**  
Cairns, July 29, 2010

# Why Bose-Bose mixtures? (Our) motivation

- ▷ Bose-Bose mixtures in optical lattices map to spin hamiltonians



J. Catani et al., Phys. Rev. A (2008)

- ▷ few-body physics in ultracold collisions:
  - Efimov resonances with heavy/light atoms [G. Barontini et al., Phys. Rev. Lett. (2009)]
  - scattering in confined dimensions [G. Lamporesi et al., Phys. Rev. Lett. (2010)]
- ▷ entropy control (and thermometry) of species A by means of a species B [J. Catani et al., Phys. Rev. Lett. (2009)]

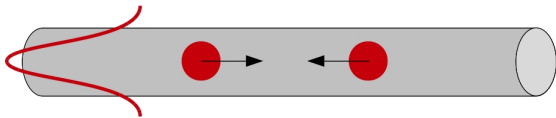
# Two-Body scattering in low dimensions

# Scattering in a waveguide

[M. Olshanii, Phys. Rev. Lett. 81, 938 (1998)]

Quick reminder: scattering of two atoms via a pseudo-potential  $U(r) = g\delta(\vec{r})(r\partial_r)$  in a tight waveguide

Strong confinement along two directions:  $E \ll \omega$ ,  $kl \ll 1$ ,  $\ell = \sqrt{\hbar/m\omega}$



- Scattering amplitude

$$f = -\frac{1}{1 + ika_{1D}}$$

- 1-dimensional scattering length

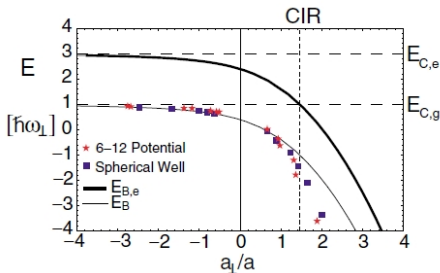
$$a_{1D} = -\frac{\ell^2}{a}\left(1 - C\frac{a}{\ell}\right), C = 1.4603/\sqrt{2}$$

- Same as 1D potential  $U(z) = g_{1D}\delta(z)$ ,  $g_{1D} = -\frac{\hbar^2}{\mu a_{1D}}$

Confinement-induced resonance (CIR):  $a_{1D} \rightarrow 0$ ,  $g_{1D} \rightarrow \infty$  for  $\ell = Ca$

# Confinement-induced resonances, interpretation

[T. Bergeman et al., Phys. Rev. Lett. 91, 163201 (2003)]



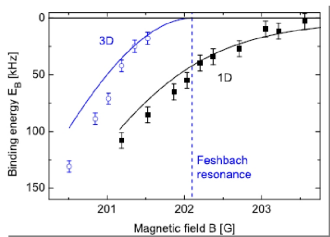
- 1D, bound state for all values of scattering length,  $a$  (vs 3D: bound state for  $a > 0$ )
- CIR as FR: “closed channel” = set of excited harmonic transverse levels
- only 1 CIR for all excited states
- decoupling of internal and center-of-mass motion

# Confinement-induced molecules and resonances, exp

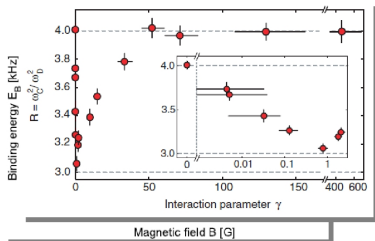
## Experiments

- Confinement-induced molecules with  $^{40}\text{K}$  atoms

[H. Moritz et al., Phys. Rev. Lett. 94, 210401 (2005)]



# Confinement-induced molecules and resonances, exp



## Experiments

- Confinement-induced molecules with  $^{40}\text{K}$  atoms

[H. Moritz et al., Phys. Rev. Lett. 94, 210401 (2005)]

- CIR on Cs atoms

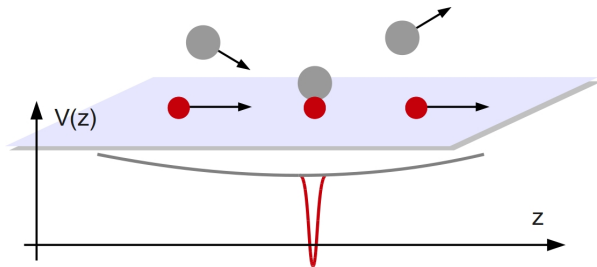
[E. Haller, Science 325, 1124 (2009)]

Very recently,

- CIR in elliptic waveguide [E. Haller et al., Phys. Rev. Lett. 104, 153203 (2010)]
- CIR in quasi-2D with  $^6\text{Li}$  atoms [P. Dyke et al., poster Mo77]

## Mix-dimensional configuration

Different kind of particles can live in different dimensions

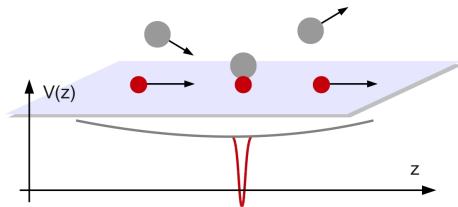


- In our experiment, two atomic species:  $^{87}\text{Rb}$  free in 3D,  $^{41}\text{K}$  confined in 2D



# Scattering resonances in mixed dimensions

[in collaboration with Y. Nishida, MIT]



Translational symmetry in the  $(xy)$  plane, drop center-of-mass  $X, Y$  coordinates

$$\left[ -\frac{\hbar^2}{2\mu} \nabla_{\mathbf{r}}^2 - \frac{\hbar^2}{2m_1} \nabla_{z_1}^2 - \frac{\hbar^2}{2m_2} \nabla_{z_2}^2 + \frac{1}{2} m_1 \omega^2 z_1^2 + gV \right] \psi(\mathbf{r}, z_1, z_2) = E \psi(\mathbf{r}, z_1, z_2),$$

where  $\mathbf{r} = (x_1 - x_2, y_1 - y_2)$

## Scattering resonances in mixed dimensions (II)

### General solution

$$\psi(\mathbf{r}, z_1, z_2) = \psi_0(\mathbf{r}, z_1, z_2) + g \int_{-\infty}^{\infty} dx' G_E(\mathbf{r}, z_1, z_2; \mathbf{0}, z', z') \psi_{reg}(z')$$
$$V\psi(\mathbf{r}_1, \mathbf{r}_2) \equiv \psi_{reg}(\mathbf{r})|_{\mathbf{r}=\mathbf{r}_1=\mathbf{r}_2} = \psi_{reg}(z)$$

At large distance  $r, |z_1 - z_2| \rightarrow \infty$ :

$$\psi(\mathbf{r}, z_1, z_2) = \phi_0(z_1) \left[ \frac{1}{a_{eff}} - \frac{1}{\sqrt{\frac{m_2}{\mu} z_2^2 + r^2}} \right], \quad (1)$$

which defines the **effective scattering length**  $a_{eff}$

At short distance  $R = \sqrt{r^2 + (z_1 - z_2)^2} \rightarrow 0$

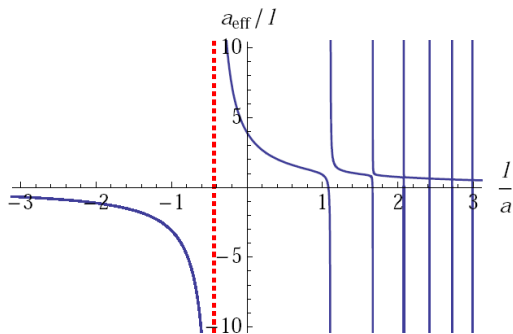
$$\psi(\mathbf{r}, z_1, z_2) = \left[ \frac{1}{a} - \frac{1}{R} \right] \psi_{reg}(z)|_{z=z_1=z_2},$$

Secular equation ties  $a_{eff}$  with  $a$ :

$$\frac{1}{a} b_i - \tilde{M}_{ij} b_j = \left[ \frac{1}{a_{eff}} \sqrt{\frac{m_2}{\mu}} \delta_{0i} \delta_{0j} + M_{ij} \right] b_j$$

## Scattering length in mixed dimensions (III)

Result:  $a_{\text{eff}} \rightarrow \infty$  for multiple values of  $a/\ell$  depending only on the mass ratio  $m_1/m_2$  (\*)



[courtesy of Y. Nishida]

Analysis very close to previous work of P.Massignan and Y.Castin, Phys. Rev. A 74, 013616 (2006)

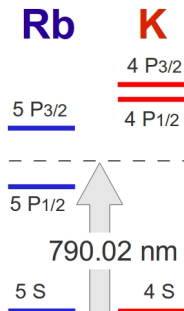
(\*) only approx with effective range corrections ...

# How-to? Species-selective dipole potential

Suitable choice of laser wavelength  $\rightarrow$  optical dipole potential selective on atomic species

[L. J. LeBlanc and J. H. Thywissen, Phys. Rev. A 75, 053612 (2007)]

For our particular mixture, i.e.  $^{87}\text{Rb}$ - $^{41}\text{K}$ ,  $\lambda = 790.02 \text{ nm}$ .



For Rb, D1 and D2 light-shifts cancel out

Tight confinement realized by 1D optical lattice

$$V_0 = sE_r$$

Array of 2D traps:

$$\ell = \lambda / (2\pi s^{1/4})$$

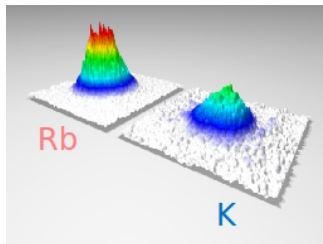
(e.g.  $\ell = 1200a_0$  for  $s = 15$ )

# Entropy transfer

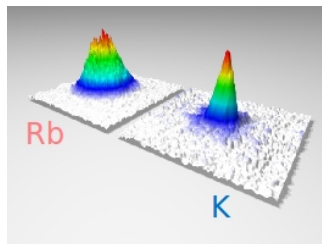
# Interspecies entropy transfer

Idea: compress only K

- $\omega(K)$ ,  $T_c(K)$  increase
- $T \sim \text{const}$  (thanks to Rb)
- $T/T_c(K)$  decreases  $\rightarrow$  K entropy lower



before



after compression

Similar to reversible BEC with “dimple” potential

[D. M. Stamper-Kurn et al., Phys. Rev. Lett. 81, 2194 (1998)]

# BEC transition

Textbook thermodynamics, ideal gases in harmonic trap  $\omega \rightarrow$  K BEC fraction

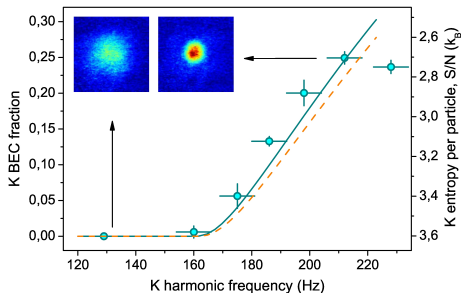
Total entropy conservation

- Above  $T_c$ :  $S/(Nk_B) = 4g_4(z)/g_3(z) - \log(z)$

with  $N = g_3(z)(T/\omega)^3$ ,  $g_n(z) = \sum_{k>0} z^k/k^n$

- Below  $T_c$ :  $S/(Nk_B) = 4[g_4(1)/g_3(1)](1 - f_c)$

BEC fraction  $f_c = 1 - t^3 - \eta[g_2(1)/g_3(1)]t^2(1 - t^3)^{2/5}$ ,  $t = T/T_c$ ,  $\eta = \mu/k_B T$

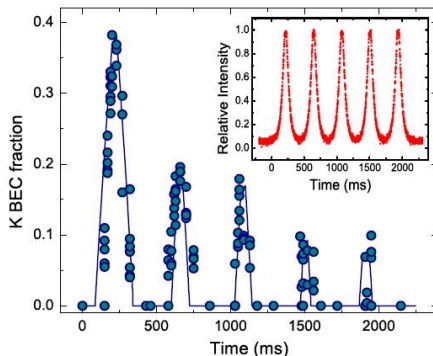


[J. Catani *et al.*, Phys. Rev. Lett. 103, 140401 (2009)]

# Reversibility of BEC phase transition

Compression/decompression cycles:  $\omega_K/(2\pi) = 128 \leftrightarrow 216$  Hz

Up to 5 cycles of BEC

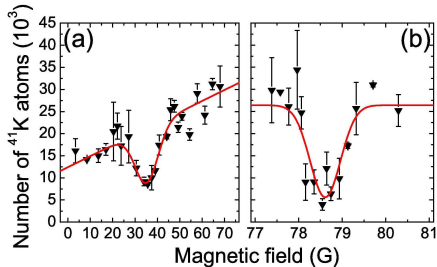


In practice, reversibility limit is heating/atom loss of Rb



# Interspecies Feshbach resonances

# Interspecies $^{41}\text{K}$ - $^{87}\text{Rb}$ Feshbach resonances



Inelastic atomic collisions, 2 interspecies  
FR:

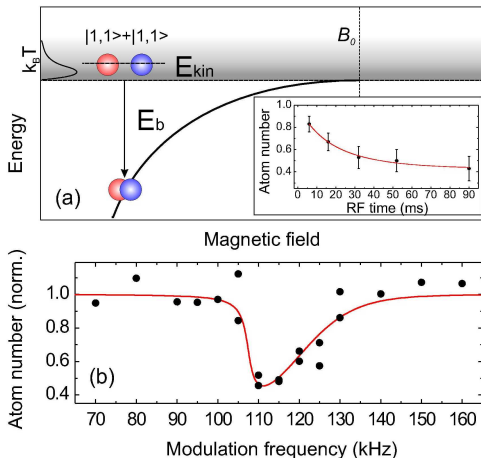
broad,  $B_0 \simeq 39\text{G}$

narrow,  $B_0 \simeq 79\text{G}$

G. Thalhammer *et al.*, Phys. Rev. Lett. 100, 210402  
(2008)

# RF association of Feshbach molecules

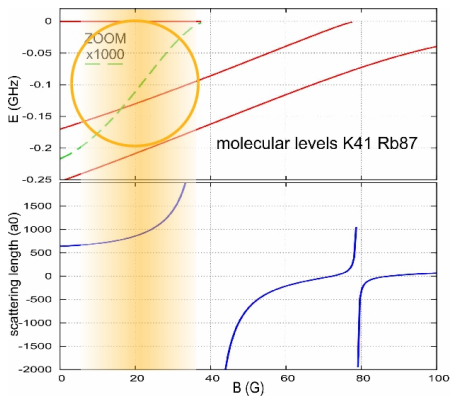
Precise method to locate FR  $\rightarrow$  molecular spectroscopy, i.e. drive free-to-bound transitions by oscillating magnetic field [S. T. Thompson *et al.* Phys. Rev. Lett. 95 190404 (2005)]



Measure loss of atoms converted in molecules

[C. Weber *et al.* Phys. Rev. A 78 061601(R) (2008)]

# Control of interspecies interactions



Feshbach spectroscopy +

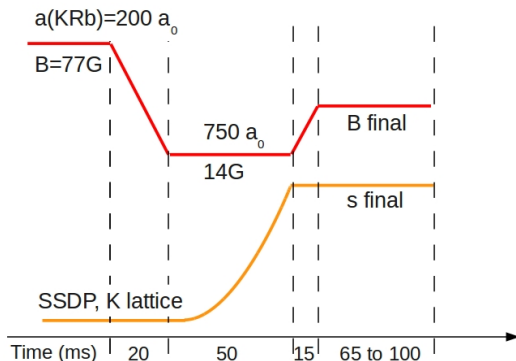
collisional model, A. Simoni,  
University of Rennes

→ knowledge of  $^{41}K$ - $^{87}Rb$   
scattering length

# Mix-dimensional scattering

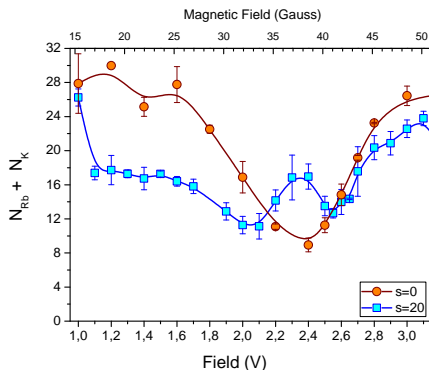
## Experiment

- Mixture in (1,1)+(1,1) hf states, sympathetic cooling in optical trap to 300 nK
  - Set B below both FR, raise SSDP lattice to selectively confine K in 2D
  - Set B to final value, hold for fixed time, record remaining atom number
- Assumption: enhancement of inelastic collisions, i.e. atom number minima, for  $a_{\text{eff}} \rightarrow \infty$



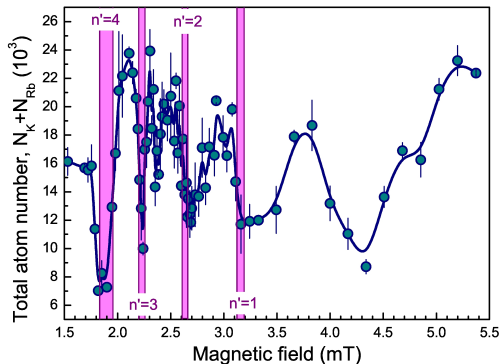
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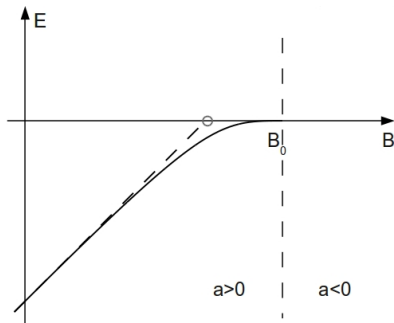


## Simple physical picture

Position of resonances obtained by a simple argument

Degeneracy:

$$E(\text{threshold}) = E \text{ of bound state}$$

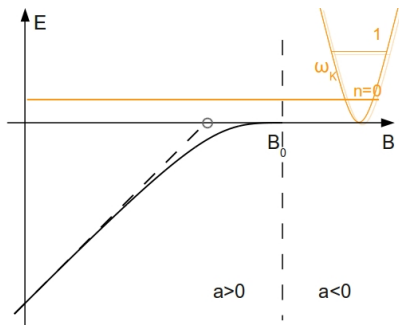


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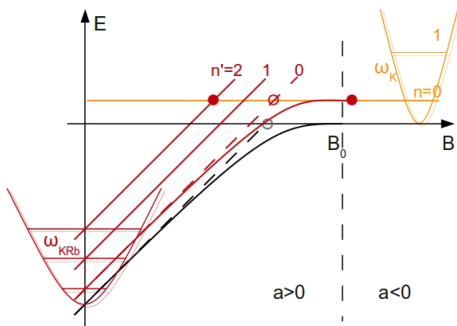
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# Simple physical picture

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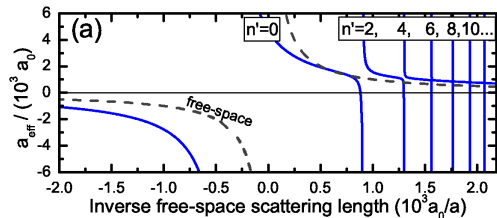
$E(\text{threshold}) = E$  of bound state from excited confined states

$$(1/2)\omega_K = (n' + 1/2)\omega_{KRb} - E_{\text{binding}}/\hbar$$

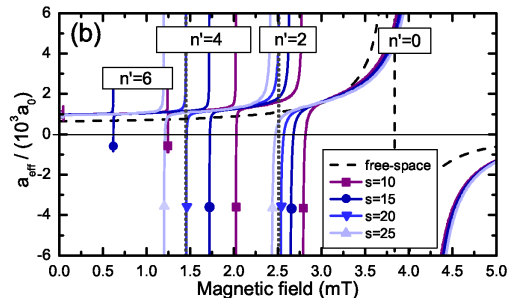
where  $\omega_{KRb} = \omega_K \sqrt{m_K/(m_K + m_{Rb})}$   
 $p_{Rb}^2/(2m_{Rb})$  neglected

- very good agreement with above results of scattering theory
- multiple resonances because internal and center-of-mass motion are coupled
- by parity conservation, only states with **even- $n'$**  couple to  **$n = 0$**  (for  $p_{Rb} = 0$ )

## Predictions for our specific FR

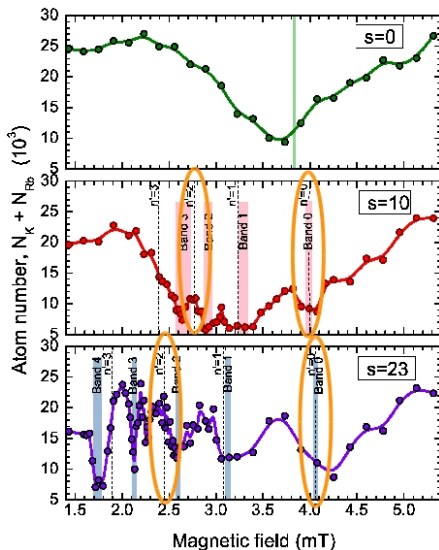


To change  $a_{KRb}$ , change magnetic field



To change  $\omega_K, \omega_{KRb}$ , change  $s$ , strength of SSDP lattice

# Experimental data



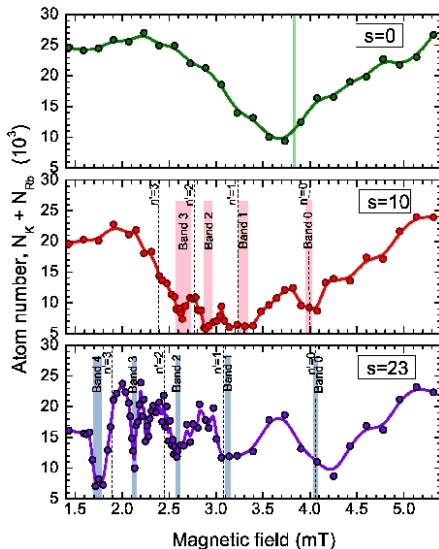
Predictions do not match the data

- positions are offset
- peaks at odd- $n'$

Harmonic oscillator levels are not correct  $\rightarrow$  lattice band structure

- At  $T = 300$  nK,  $\sqrt{k_B T / m_K} \simeq 0.6 \hbar k \rightarrow$  thermal filling of 1st BZ
- quasi-momenta  $q \neq 0$  not parity-eigenstates  $\rightarrow$  odd- $n'$  peaks allowed

# Experimental data



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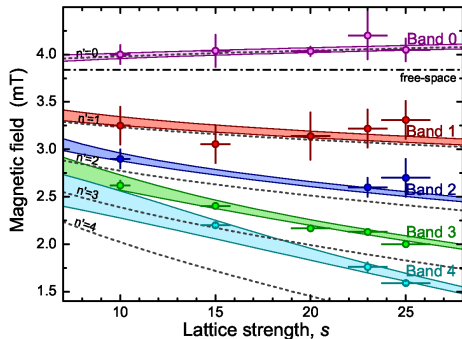
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# Experiment/theory comparison

Energy degeneracy re-calculated:

$$p^2/(2m_{Rb}) + \epsilon_K(0, q; V_{\text{lat}}^K) = \epsilon_{KRb}(n', q + p; V_{\text{lat}}^K) - E_b$$



- $\epsilon_i(n, q; V_{\text{lat}}^K)$ , energy of the Bloch wave of particle  $i = K, KRb$
- $(n, q)$  quasimomentum/band index
- $V_{\text{lat}}^K$  lattice potential
- $p$  initial Rb momentum
- $E_b$ , binding energy

Neglect resonance shift due to channel coupling, for all  $n' > 0$

## Summary

- ▷ species-selective potential engineers a mix-dimensional configuration
- ▷ entropy transfer and reversible BEC
- ▷ mix-dimensional scattering resonances are observed, their position is accounted for by simple energy argument
- ▷ lattice band structure needed
- ▷ mix-dimensional configuration a route to Efimov physics?  
Identical particles:  $2.3 < D < 3.8 \Rightarrow D = 3$   
2 fermions:  $m_1/m_2 > 13.6$  in 3D,  $m_1/m_2 > 6.35$  in 2D-3D,  $m_1/m_2 > 2.06$  in 1D-3D



# Acknowledgements

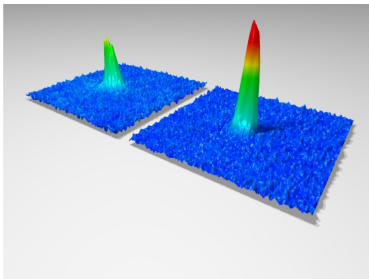
## BEC3 Group at LENS, Firenze



Staff: M. Inguscio, FM  
Postdocs: J. Catani, G. Lamporesi,  
G. Thalhammer (now in Innsbruck)  
PhD students: G. Barontini, C. Weber (now in Bonn)  
Undergraduate students: F. Rabatti

**Positions available for PhD students**

The end



Thank you

<http://quantumgases.lens.unifi.it>