# HMS Engineering Mathematics 1

Lecture 2 - Vectors

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# What are we going to learn ?

Perform simple calculations with vectors in 2-D and 3-D, including dot and cross products

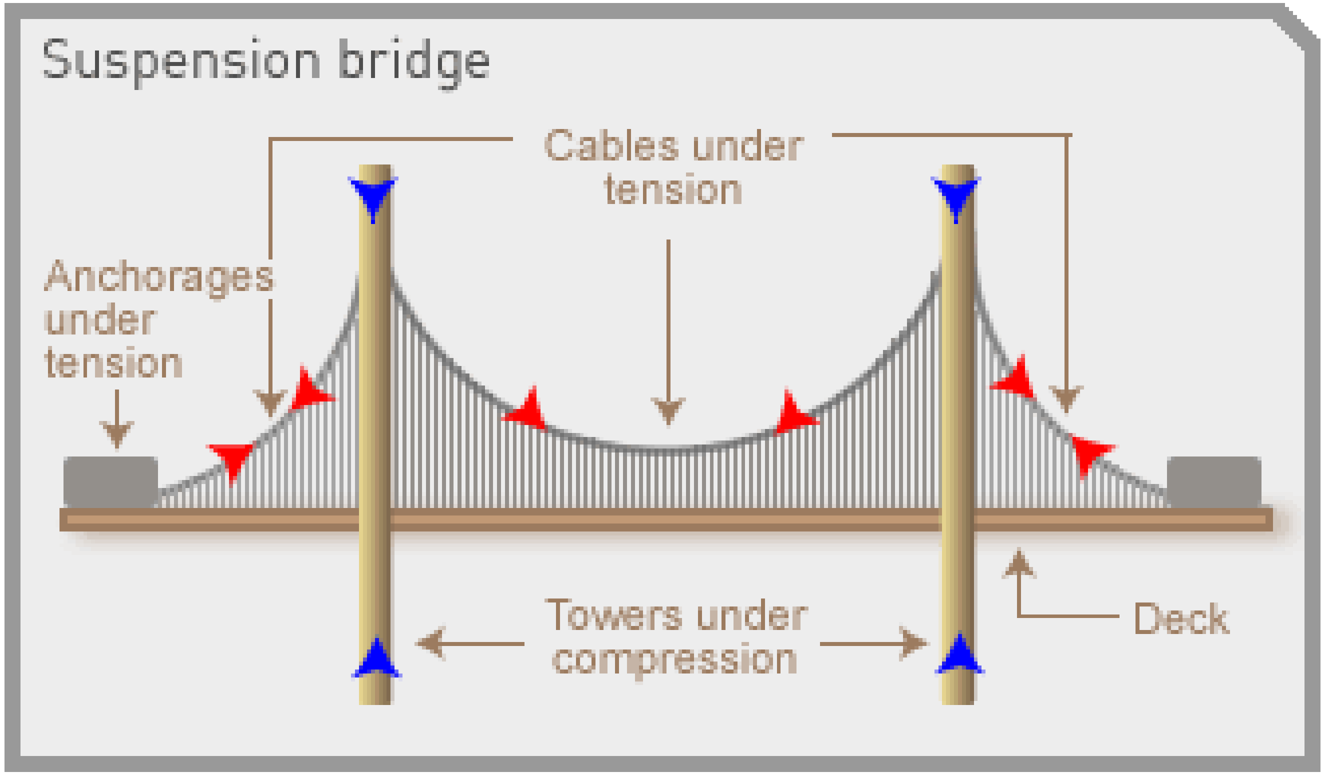
Chapter 1 of Student Notes

# Time to think

How much stress do the structural members of a bridge experience ?

Picture c/o www.makingthemodernworld.org.uk

# Suspension Bridges




Picture c/o www.makingthemodernworld.org.uk

# Vectors ?

In real world physics, we can't ignore direction.

Vectors are quantities that have both magnitude and direction

e.g. force exerted by the wind on the sections of a bridge.

Scalar quantities are quantities that only have magnitude

e.g. length, temperature, angle, time, mass, etc.

# First Problem

Abel Tasman (maximum speed of 20km/hr) needs to travel due north but current is flowing south east at 20km/hr.

(i) Actual direction if boat was simply pointed north

(ii) The course required to head north.

(iii) The ships velocity relative to Point Lonsdale

# Step 1 – Draw a diagram

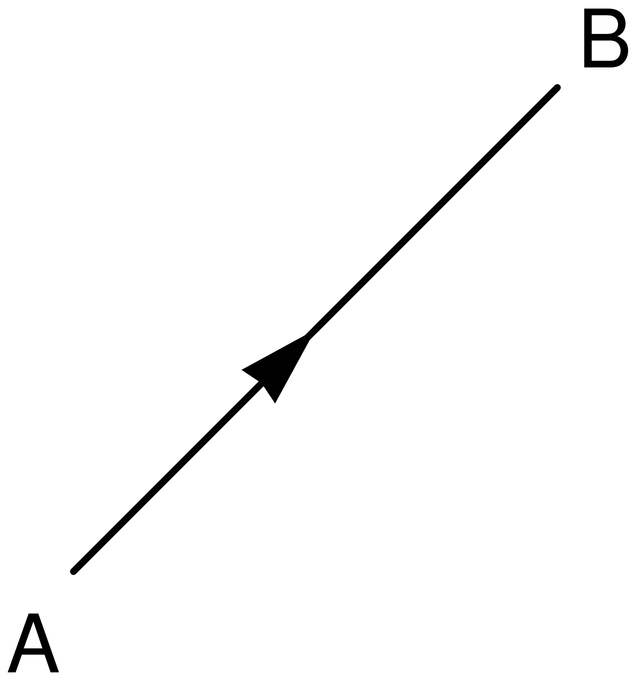
Abel Tasman (maximum speed of 20km/hr) needs to travel due north but current is flowing south east at 20km/hr.

(i) Actual direction if boat was simply pointed north

(ii) The course required to head north.

(iii) The ships velocity relative to Point Lonsdale

# Representing Vectors



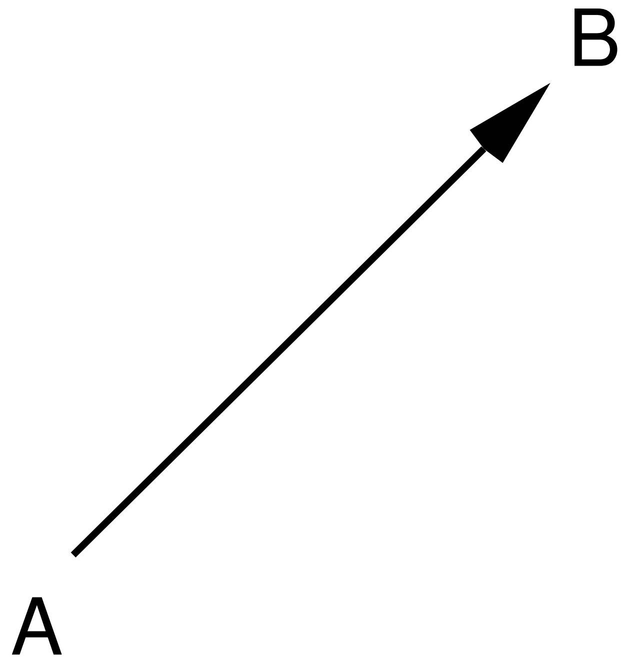




Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Vector Symbols

$\underset{\sim}{a}$ or $\overset{\ast}{AB}$ or **a** or **AB** or $\overline{AB}$ or $\overline{a}$

with || around the above to indicate magnitude

I will use mainly $\overset{\ast}{AB}$ or **a** for vector

# Direction and magnitude

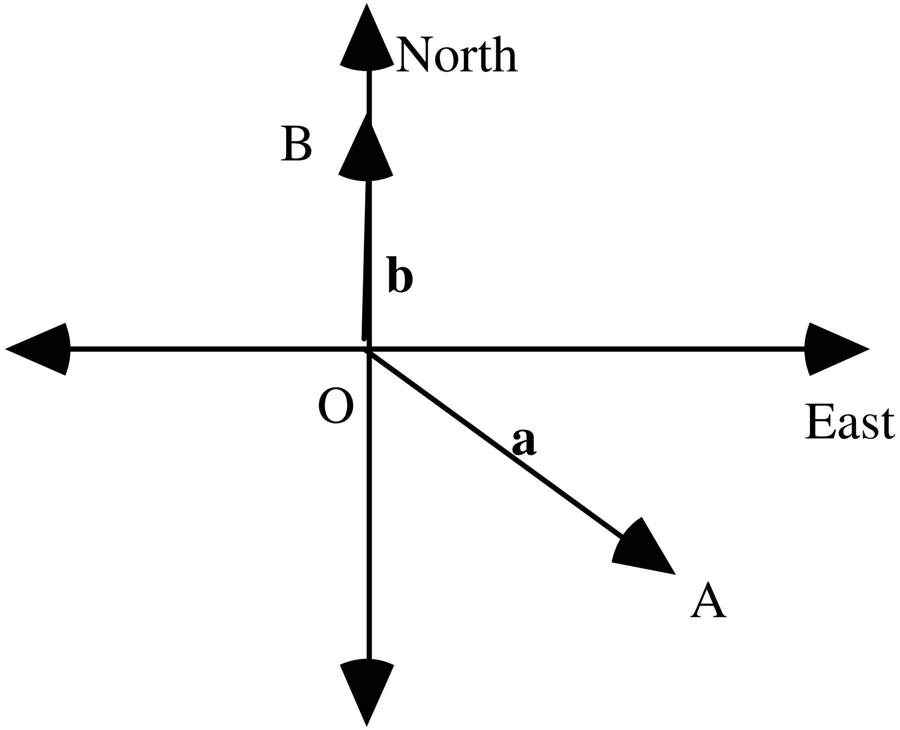



Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Which direction ?

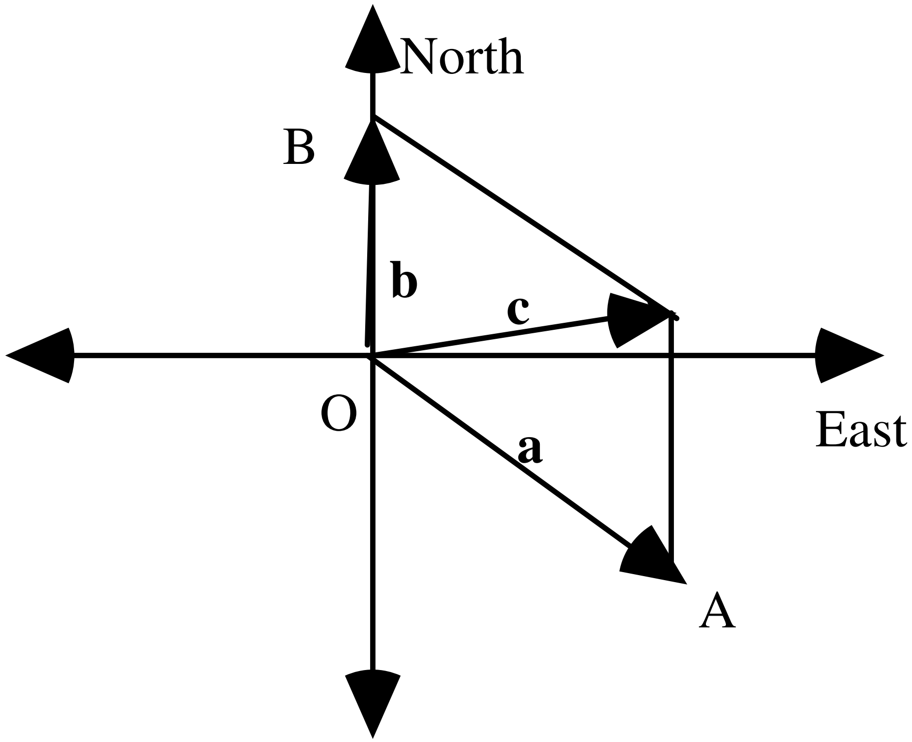
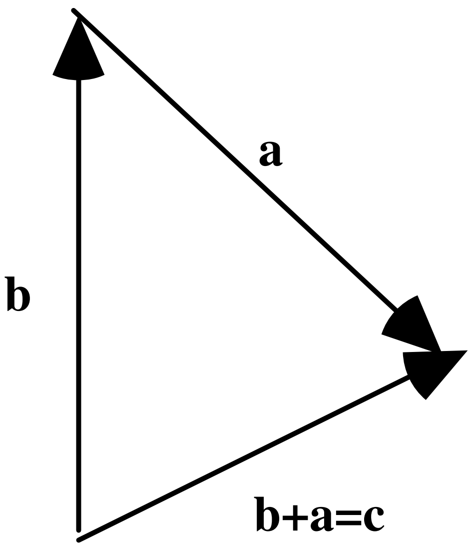
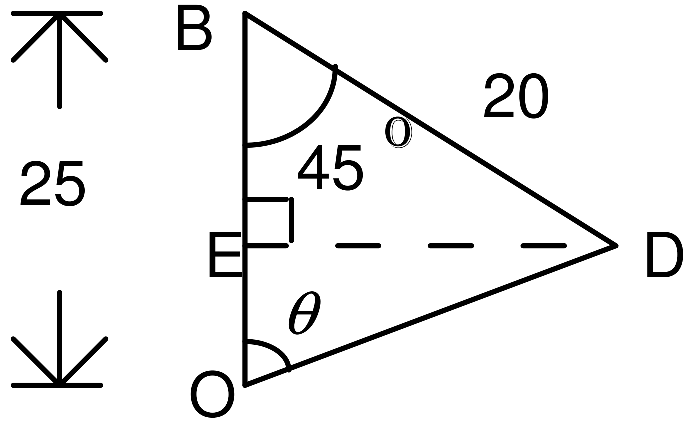



Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Head to tail rule for vector addition

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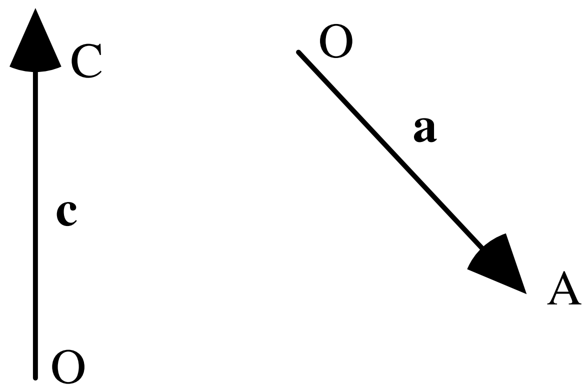
# Which way to arrive north ?

Let **OC** = **c** = desired direction of travel

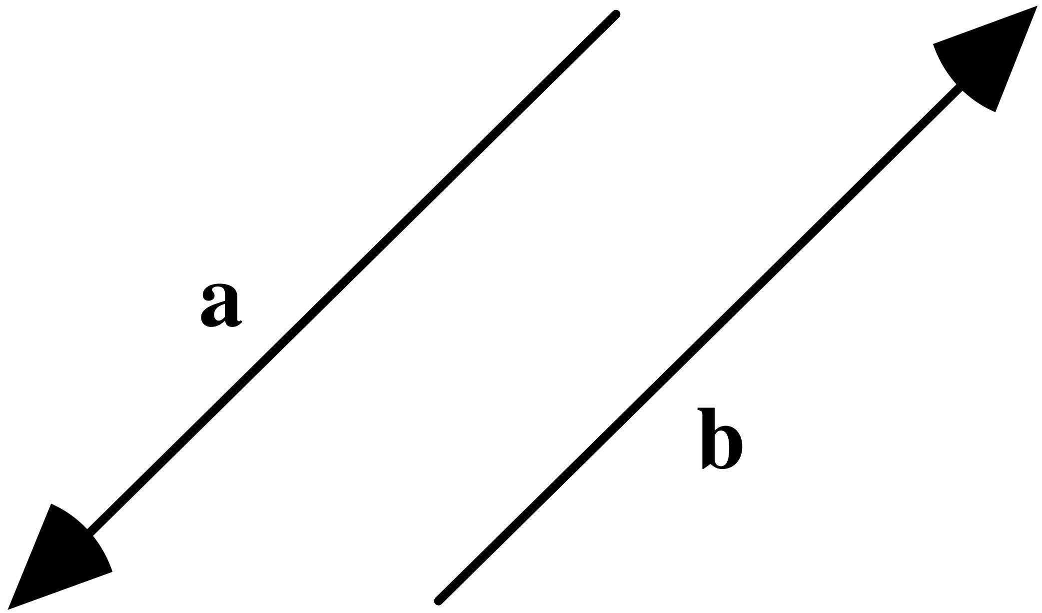



**b**= velocity of the ship relative to the water

We know **a** + **b** = **c**  therefore b = **-a** + **c**

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Rules on vectors of different directions

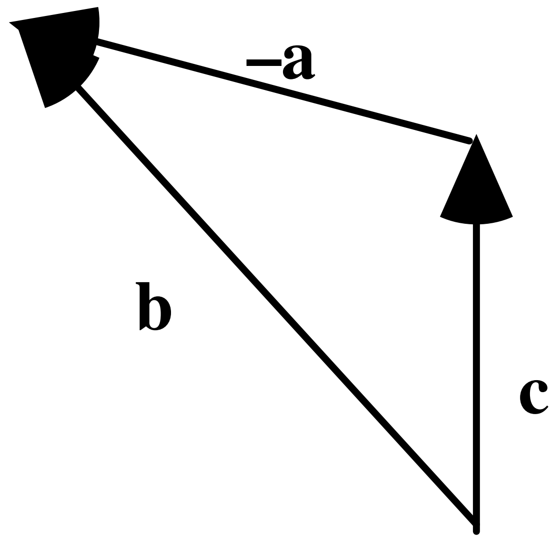



**a = -b**

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Subtraction of vectors

Subtracting is same as adding negative




**b = c - a**

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Sine Rule

The sides of a triangle are proportional to the sines of the opposite angles

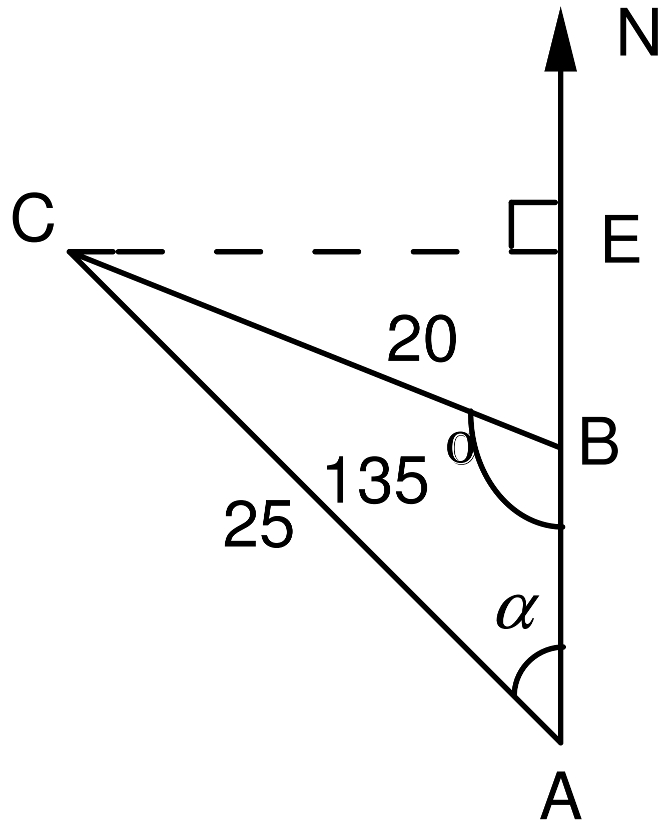
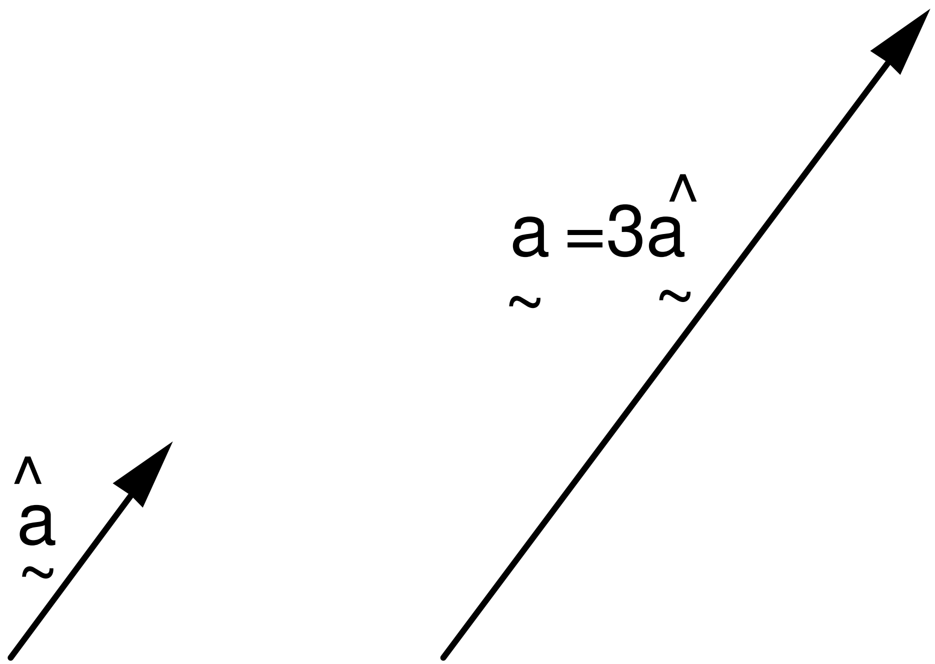



Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

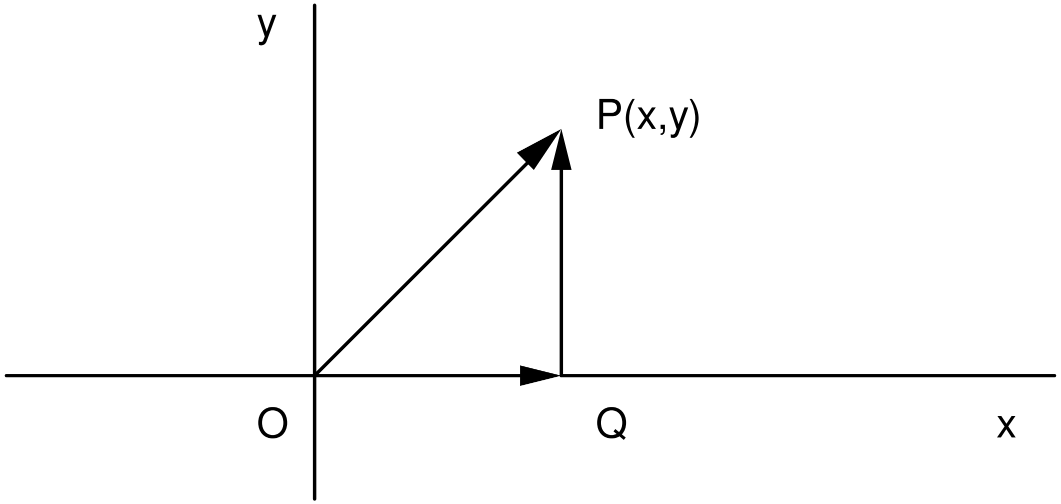
# Unit Vectors




Unit Vector has length of 1

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# 2D Unit Vectors




$$\vec{OP} = \vec{OQ} + \vec{QP}$$

$$\underset{\sim}{r} = x\,\underset{\sim}{i} + y\,\underset{\sim}{j}$$

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Vector addition using unit vectors

Given that $\vec{OA} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\vec{OB} = 5\underset{\sim}{i} - 3\underset{\sim}{j}$, find $\vec{AB}$ and the length $AB$.

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\Rightarrow \vec{AB} = \vec{OB} - \vec{OA}$$

$$\Rightarrow \vec{AB} = 5\underset{\sim}{i} - 3\underset{\sim}{j} - (3\underset{\sim}{i} + 2\underset{\sim}{j})$$

$$\Rightarrow \vec{AB} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$$

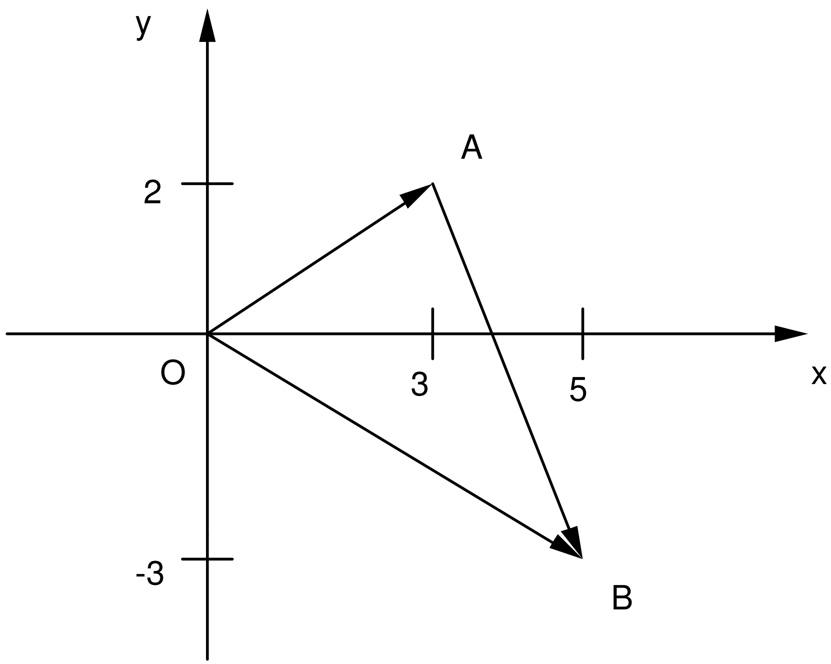
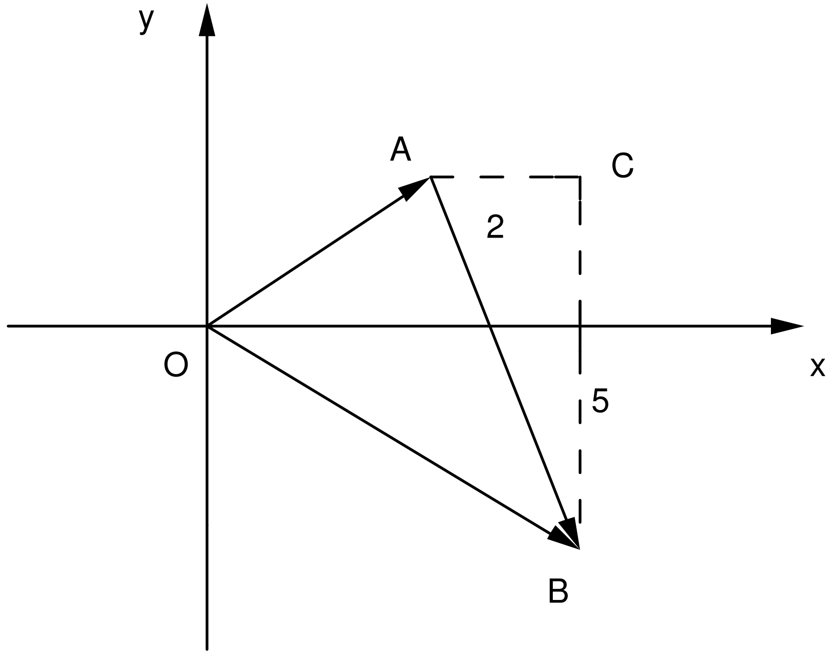



Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Finding the magnitude




$\Rightarrow$

$$AB^2 = AC^2 + CB^2 \quad \Rightarrow \quad AB^2 = 2^2 + 5^2 \Rightarrow \quad AB = \sqrt{29}$$

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# 3D Unit Vectors

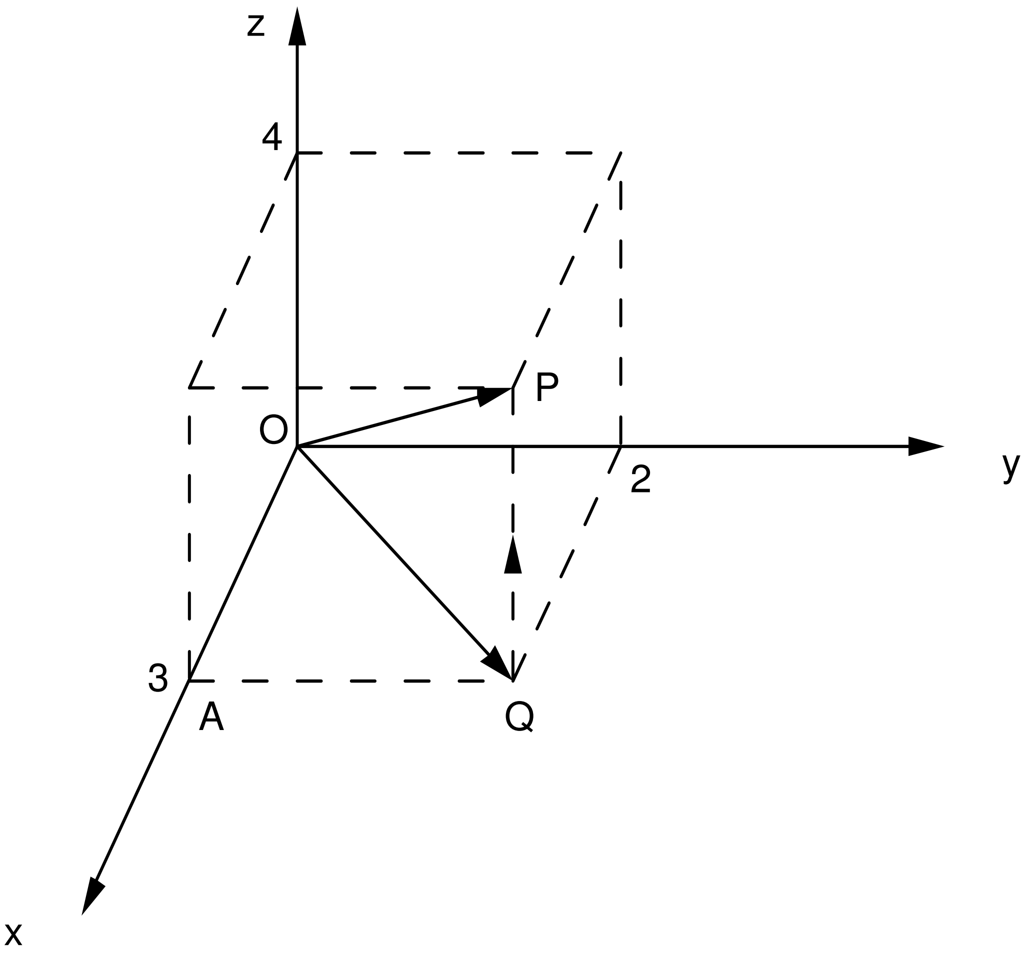



Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# 3-D Unit Vectors

In general, given a vector $\overrightarrow{OP} = \underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

then the length of $\underset{\sim}{r} = r = |\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2}$

The vector $\underset{\sim}{r}$ is called the position vector of the point $P(x, y, z)$.

The vector $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ is sometimes written in the form $\underset{\sim}{r} = (x, y, z)$.

# Direction Cosine

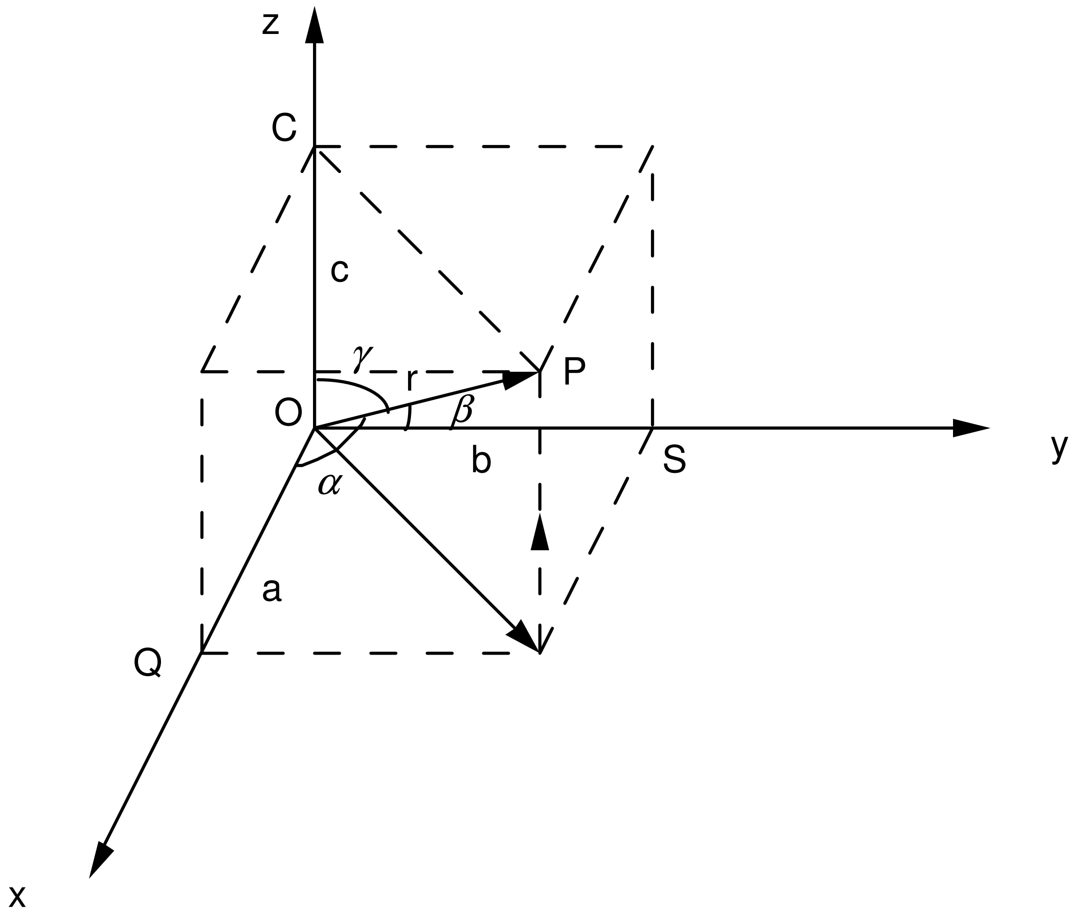
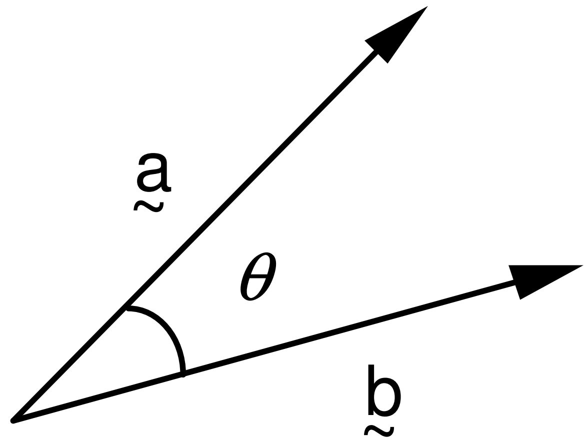



Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Dot (Scalar) Products

$$\underset{\sim}{a}.\underset{\sim}{b} = ab\cos\theta$$

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# Dot (Scalar) Product

If $\underset{\sim}{a} = a_1 \underset{\sim}{i} + a_2 \underset{\sim}{j} + a_3 \underset{\sim}{k}$ and $\underset{\sim}{b} = b_1 \underset{\sim}{i} + b_2 \underset{\sim}{j} + b_3 \underset{\sim}{k}$,

$$\underset{\sim}{a} . \underset{\sim}{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

# Dot (Scalar) Products

**Example:**

Given that $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + 1\underset{\sim}{j} + 6\underset{\sim}{k}$

find (i) $\underset{\sim}{a} . \underset{\sim}{b}$ (ii) $a = \left|\underset{\sim}{a}\right|$ (iii) $b = \left|\underset{\sim}{b}\right|$

(iv) $\cos\theta$, where $\theta$ is the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ (v) $\theta$

**Answer:**

(i) Using (4), $\underset{\sim}{a} . \underset{\sim}{b} = (2)(4) + (3)(1) + (5)(6) = 8 + 3 + 30 = 41$

(ii) $a = \left|\underset{\sim}{a}\right| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$

(iii) $b = \left|\underset{\sim}{b}\right| = \sqrt{4^2 + 1^2 + 6^2} = \sqrt{53}$

(iv) (1) $\Rightarrow \cos\theta = \dfrac{\underset{\sim}{a} . \underset{\sim}{b}}{ab} = \dfrac{41}{\sqrt{38}\sqrt{53}}$

(v) $\Rightarrow \theta = \cos^{-1}\left(\dfrac{41}{\sqrt{38}\sqrt{53}}\right) = 24^0 00'$

# Work Done

Work done = force in a particular direction causes a displacement in a particular direction

$$W = \underset{\sim}{F}.\underset{\sim}{D}$$

$$= F.D \cos\theta$$

Where $\theta$ = the angle between the direction of the force and the line of displacement

Work done = a scalar product of two vectors

# Work Done

**Example:**
A force $\underset{\sim}{F} = 5\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$ newton acts on a particle which moves from A(1,7,11) to B(4,16,18) (see Fig. 13), with the coordinates in metres. Find $W$, the work done by the force on the particle, where $W$ is defined by $W = \underset{\sim}{F} . \overrightarrow{AB}$.

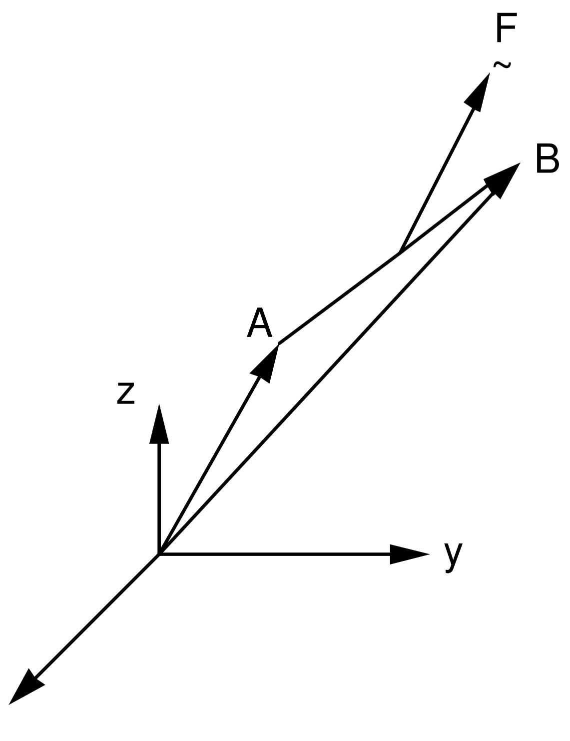


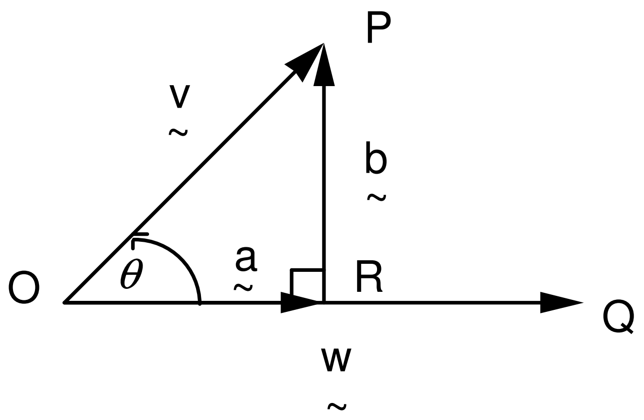

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Vector Components

Consider the vectors $\underset{\sim}{v}$ and $\underset{\sim}{w}$ as in Fig. 14 (see below)

with $\underset{\sim}{v} = \overrightarrow{OP}$ and $\underset{\sim}{w} = \overrightarrow{OQ}$. The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are defined by

$\underset{\sim}{a} = \overrightarrow{OR}$ and $\underset{\sim}{b} = \overrightarrow{RP}$.




The vector $\underset{\sim}{a}$ is called the (vector) component of $\underset{\sim}{v}$ in the $\underset{\sim}{w}$-direction,

$a$ is called the scalar component of $\underset{\sim}{v}$ in the $\underset{\sim}{w}$-direction, and

$\underset{\sim}{b}$ is called the component of $\underset{\sim}{v}$ perpendicular to the $\underset{\sim}{w}$- direction.

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

# Archimedes (287-212 BC)

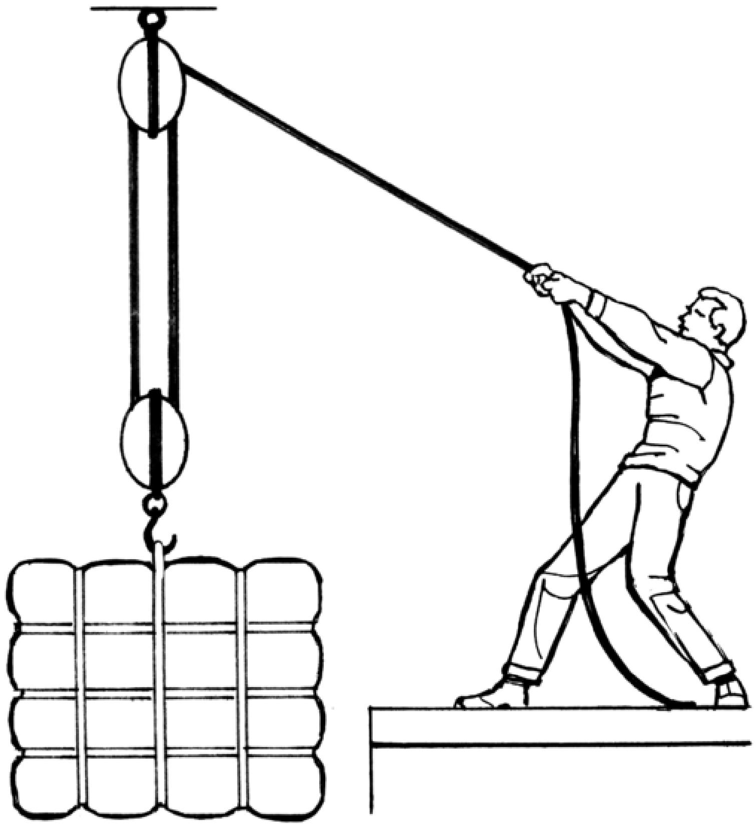
Brilliant engineer, mathematician, physicist & inventor

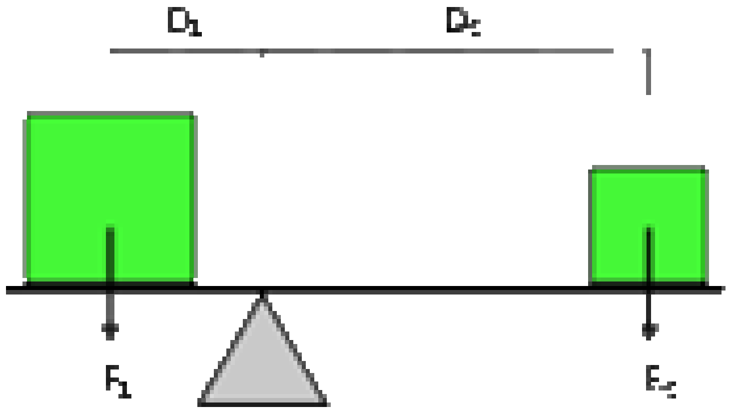
- Integral calculus, summation on infinite series, areas of parabolas and volume of spheres
- Principles of levers, pulleys and pumps
- Pioneer in concentration of solar energy

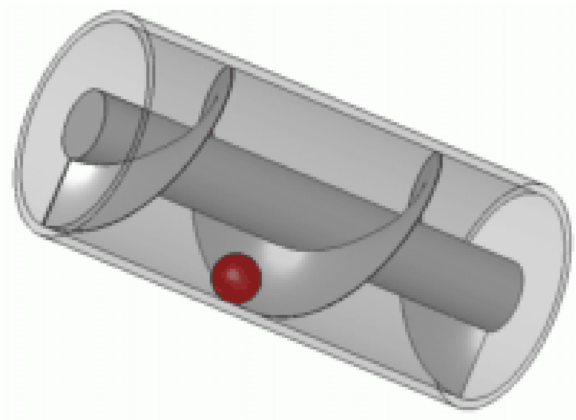
*Archimedes Thoughtful* by Fetti (1620)

"Give me a place to stand and I will move the earth"

# Archimedes (287-212BC) – Force at a Distance







Images from wikipedia

# Vector Cross Product

If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are two vectors, with $\theta$ the angle between them (see Fig. 12), then the vector (cross) product of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is written as $\underset{\sim}{a} \times \underset{\sim}{b}$ and is defined by

$$\underset{\sim}{a} \times \underset{\sim}{b} = (ab \sin\theta)\underset{\sim}{\hat{n}}$$

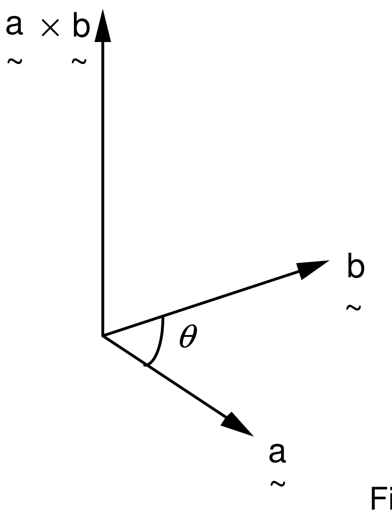



Fig. 15

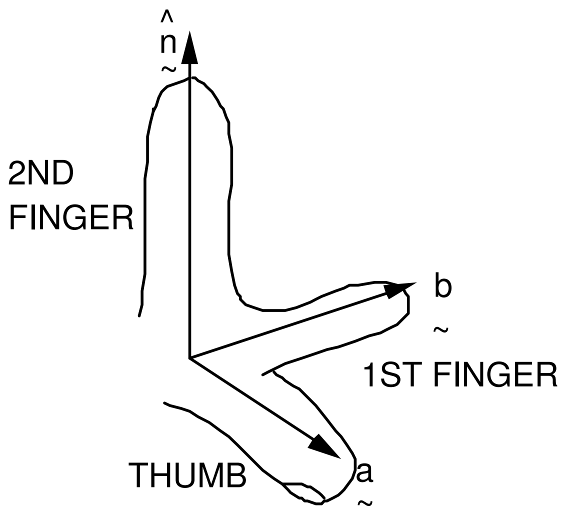



Fig.16

Image from Barling et al. HMS111 Engineering Mathematics, Student Notes, 2009

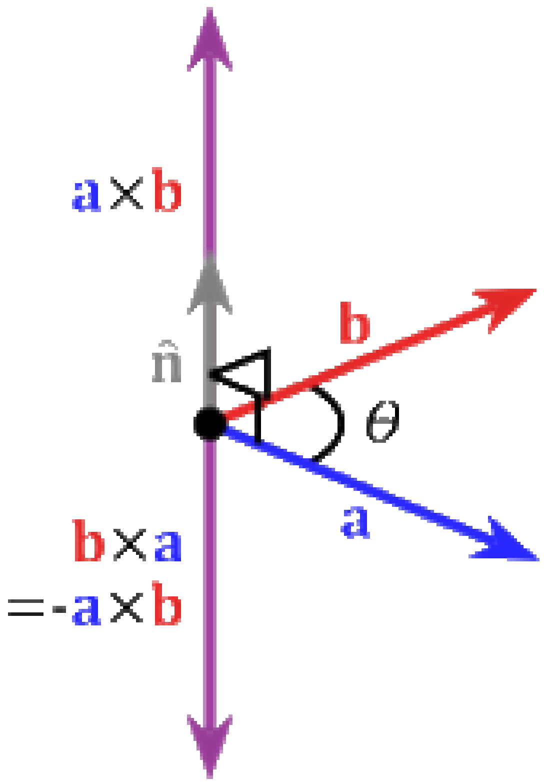
# Cross Product

$$\Rightarrow \underset{\sim}{a} \times \underset{\sim}{b} = (a_2 b_3 - a_3 b_2)\,\underset{\sim}{i} + (a_3 b_1 - a_1 b_3)\,\underset{\sim}{j} + (a_1 b_2 - a_2 b_1)\,\underset{\sim}{k}$$

Cross Product results in a vector quantity

Dot Product results in a scalar quantity

# Cross Product



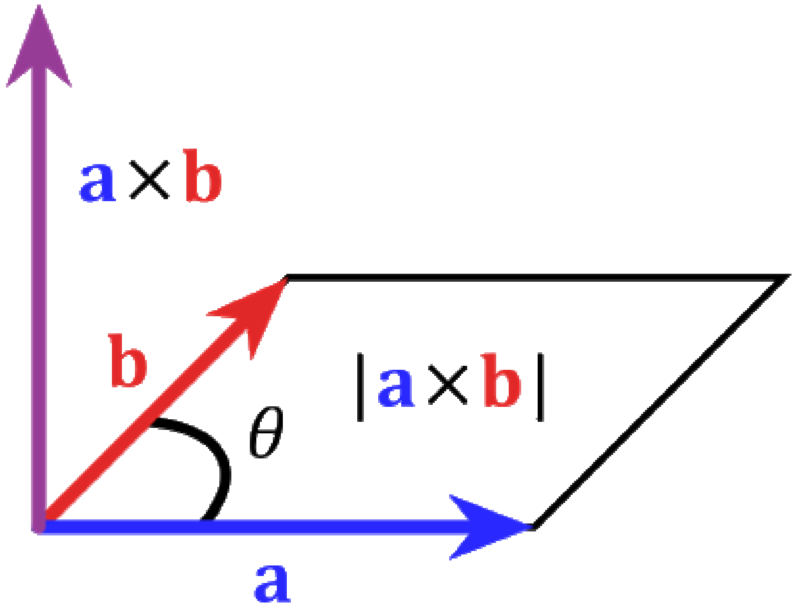
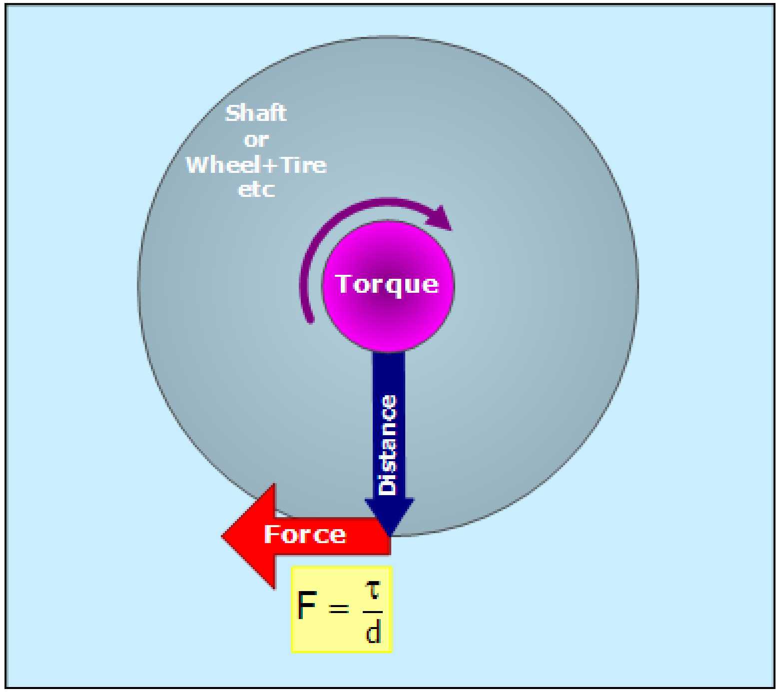
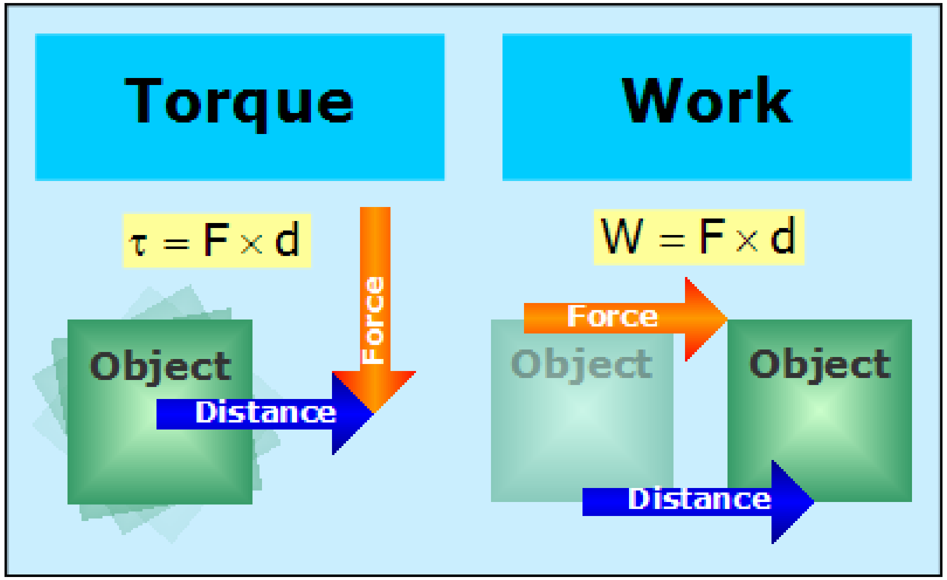




Image from wikipedia

# Force at a Distance - Torque




http://craig.backfire.ca/pages/autos/horsepower

# Force At A Distance - Pulleys

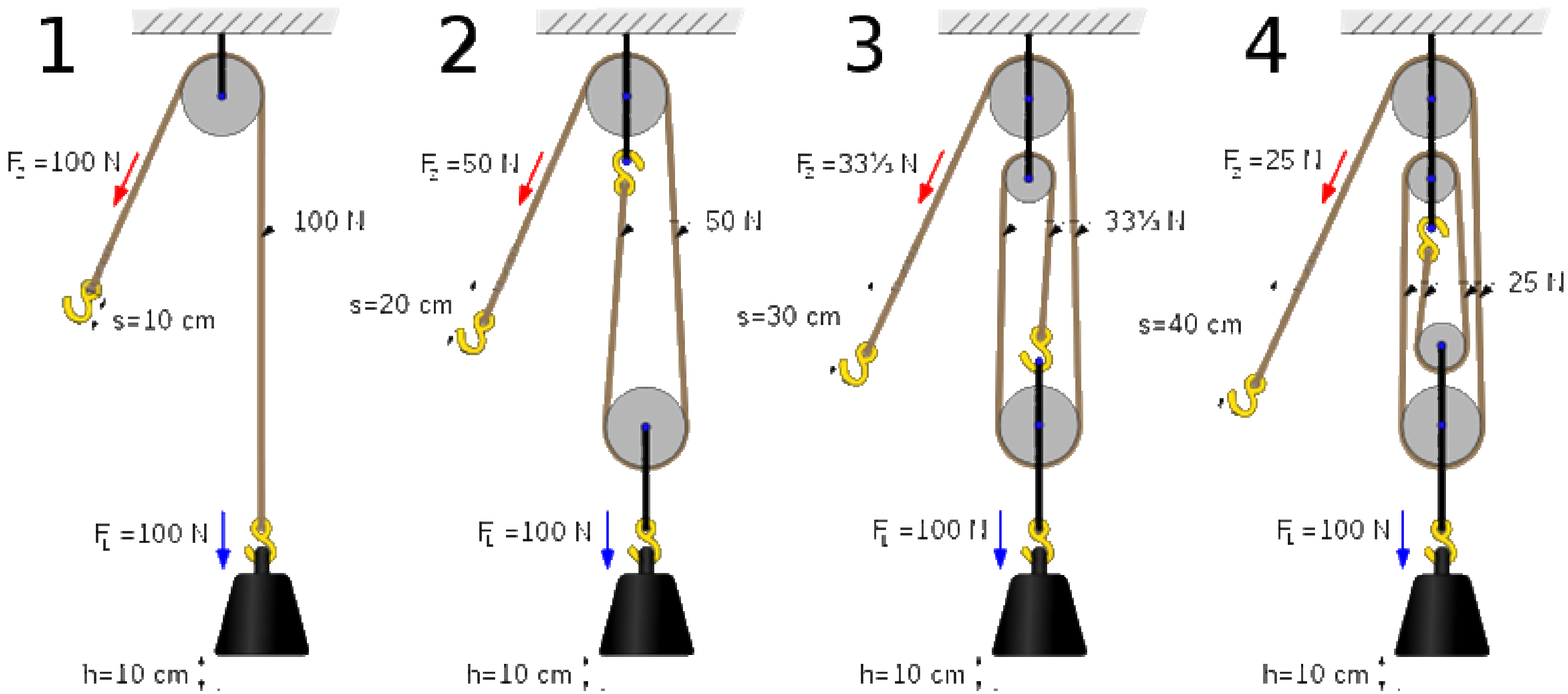



Image from wikipedia

# Force At A Distance - Beams

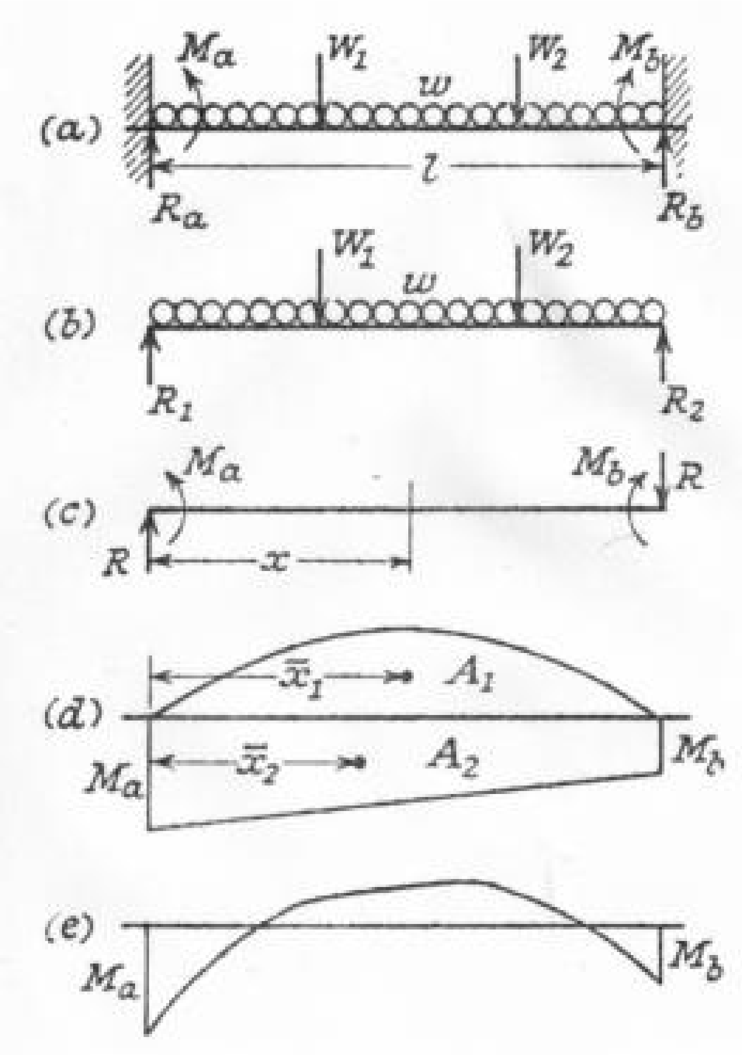



Image from www.codecogs.com and www.pbs.org