

HMS111

Engineering Mathematics 1

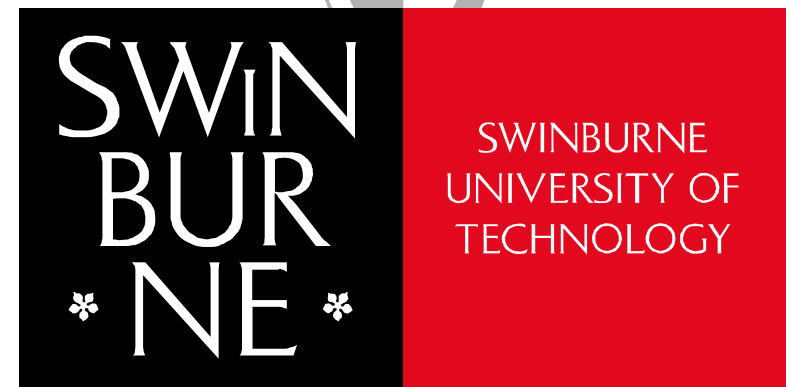
Numbers

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What are we going to learn ?

Do calculations involving binary, octal and hexadecimal numbers

Why do we count in 10s ?

- Decimal system reflects human characteristics not an innately better system of counting

e.g. an imperial “foot” (12 inches) can be readily divided by 1,2,3,4,6 and 12 to form an integer

- Machines don't have ten fingers !
- Binary (digital circuitry), octal (unix/mainframe) and hexademinal (html&URL) systems are important in computers

Claude Shannon – Founder of Modern Circuitry

Electrical Eng. & Mathematician

(1916-2001)

Combine Boolean Logic/Binary

(Master Thesis 1938)

Computer Chess & AI

Communication Theory



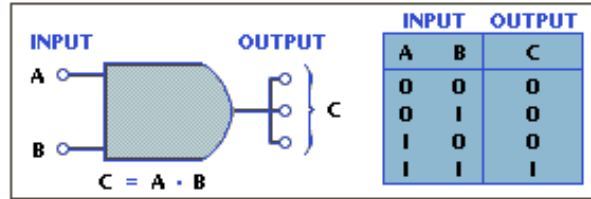
Photo from wikipedia

Motorized pogo sticks, roulette computer and flame flowing trumpets !!

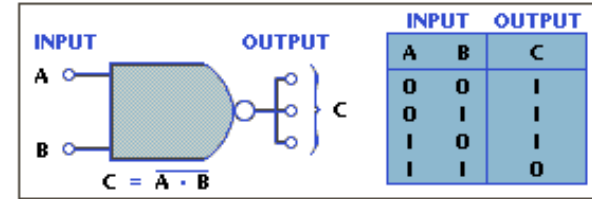
Ideas are evident in every computer and microprocessor

Digital Circuits

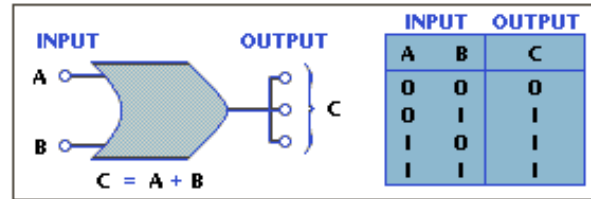
AND (A and B) GATE



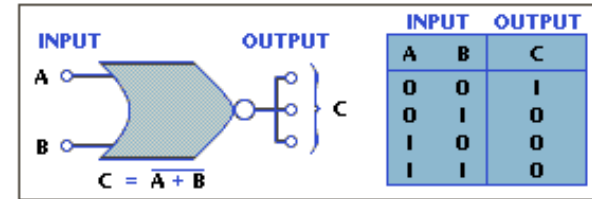
NAND (not A and B) GATE



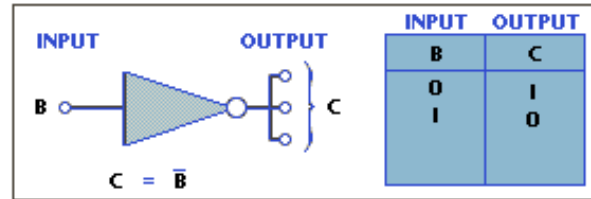
OR (A or B or both) GATE



NOR (not A or B or both) GATE



NOT (not C) GATE



EXCLUSIVE OR (A or B but not both) GATE

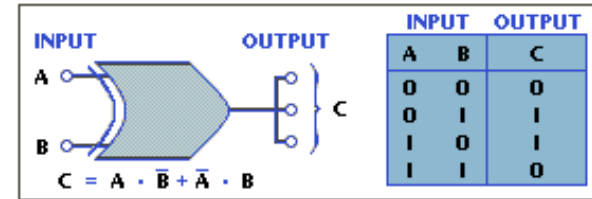


Image from Encarta.msn.com

Recommend: R.P. Feynman "Feynman Lectures on Computation"

Decimal

258.1 is shorthand for

$$2 \times 10^2 + 5 \times 10^1 + 8 \times 10^0 + 1 \times 10^{-1}$$

“Base 10” means ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

In general,

if r is the base and a_j are digits that range from 0 to $r-1$

$$a_n \times r^n + a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} \dots$$

Decimal

How do we add in decimals ?

What are the rules ?

Can you generalize those rules ?

Binary

110.1 is shorthand for

$$1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1}$$

Adding in binary:

$$\begin{aligned} 101.1 + 11.1 &= (0.1 + 0.1) + (1 + 1) + (0 + 10) + (100 + 0) \\ &= (0.0 + 1.0) + (0.0 + 10) + 10 + 100 \\ &= 1.0 + 100 + 100 = 1001 \end{aligned}$$

Binary

Adding in binary:

$$\begin{aligned}101.1 + 11.1 &= (0.1 + 0.1) + (1 + 1) + (0 + 10) + (100 + 0) \\&= (1/2 + 1/2) + (1 + 1) + (0 + 2) + (4 + 0) = 9_{10} \\&= (0.0 + 1.0) + (0.0 + 10) + 10 + 100 \\&= 1.0 + 100 + 100 = 1001_2 \\&= 1 \times 2^3 + \dots + 1 \times 2^0 = 9_{10}\end{aligned}$$

Binary

Lets try something a bit harder

$$110111.1_2 + 11111.01_2$$

- (i) Carry out as a straight binary operation
- (ii) Convert each number into a decimal
- (iii) Convince yourself that (i) is correct

Binary

Converting from decimal to binary ?

Keep dividing by 2

$$\begin{aligned}17_{10} &= (2 \times 8) + 1 \\ &= (2 \times 2 \times 4) + 1 \\ &= (2 \times 2 \times 2 \times 2) + 1 \\ &= 1 \times 2^4 + 1 \times 2^0 = 10001_2\end{aligned}$$

Decimal to Binary

$$\begin{aligned}157_{10} &= 2 \times 78 + 1 \\ &= 2 \times (39 \times 2) + 1 \\ &= 2 \times 2 \times ((19 \times 2) + 1) + 1 \\ &= 2 \times 2 \times 2 \times (18 + 1) + 2^2 + 1 \\ &= 2^3 (18) + 2^3 + 2^2 + 1 \\ &= 2^3 (2 \times 9) + 2^3 + 2^2 + 1 \\ &= 2^4((2 \times 4) + 1) + 2^3 + 2^2 + 1 \\ &= 2^4 + 2^7 + 2^3 + 2^2 + 1 = 100011101_2\end{aligned}$$

Binary/Decimal

$$313_{10} + 131_{10} = ?$$

- (i) Convert to a binary operation
- (ii) Confirm that the binary answer is right

Multiplying in binary

1001 x 10.1

$$\begin{array}{r} 1001.00_2 \\ 10.10_2 \\ \hline 10.10 \\ 000.00 \\ 0000.00 \\ 10100.00 \\ \hline 10110.10_2 \end{array}$$

9_{10}
 2.5_{10}

22.5_{10}

Hexadecimal and Octal

Octal used in old computer processors

Hexadecimal used to save space in memory (html)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

1 2 3 4 5 6 7 10 11 12 13 14 15 16 17 20 21 22

1 2 3 4 5 6 7 8 9 A B C D E F 10 11 12

Converting Hexadecimal to Decimal

$$C6A3.C7_{16}$$

$$= 12 \times 16^3 + 6 \times 16^2 + 10 \times 16^1 + 3 \times 16^0 + 12 \times 16^{-1} + 7 \times 16^{-2}$$

$$= 49152 + 1536 + 160 + 3 + 0.75 + 0.02734375$$

$$= 50851.77734_{10}$$

Converting decimal to hexadecimal

2411_{10} Divide by 16

$$150 \times 16^1 + 11$$

$$(9 \times 16 \times 16) + (6 \times 16) + 11$$

$$(9 \times 16^2) + (6 \times 16^1) + B$$

$$= 96B_{16}$$

Playing with Hexadecimal

$$CD_{16} + A74_{16}$$

- i) Carry out addition in hexadecimal
- ii) Convert all three numbers to decimal and prove the result to yourself

Playing with Octal

$$756_8 + 4351_8$$

- i) Carry out addition in Octal
- ii) Convert all three numbers to decimal and prove the result to yourself

Changing base for numbers less than 1

Convert 0.56 to binary

Multiple by 2

$$0.56 \times 2 = 1.12$$

$$0.56 = 1 \times 2^{-1} + 0.12 \times 2^{-1}$$

$$(0.12 \times 2 \times 2 \times 2 \times 2 = 1.92)$$

$$= 1 \times 2^{-1} + 1 \times 2^{-5} + 0.92 \times 2^{-5}$$

$$(0.92 \times 2 = 1.84)$$

$$= 1 \times 2^{-1} + 1 \times 2^{-5} + 1 \times 2^{-6} + 0.84 \times 2^{-6} \text{ etc etc}$$