HMS111
Engineering Mathematics 1

Algebra

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What are we going to learn ?

Solve quadratic equations for real roots

Solve cubic equations

Solve inequalities

Evaluate formulae

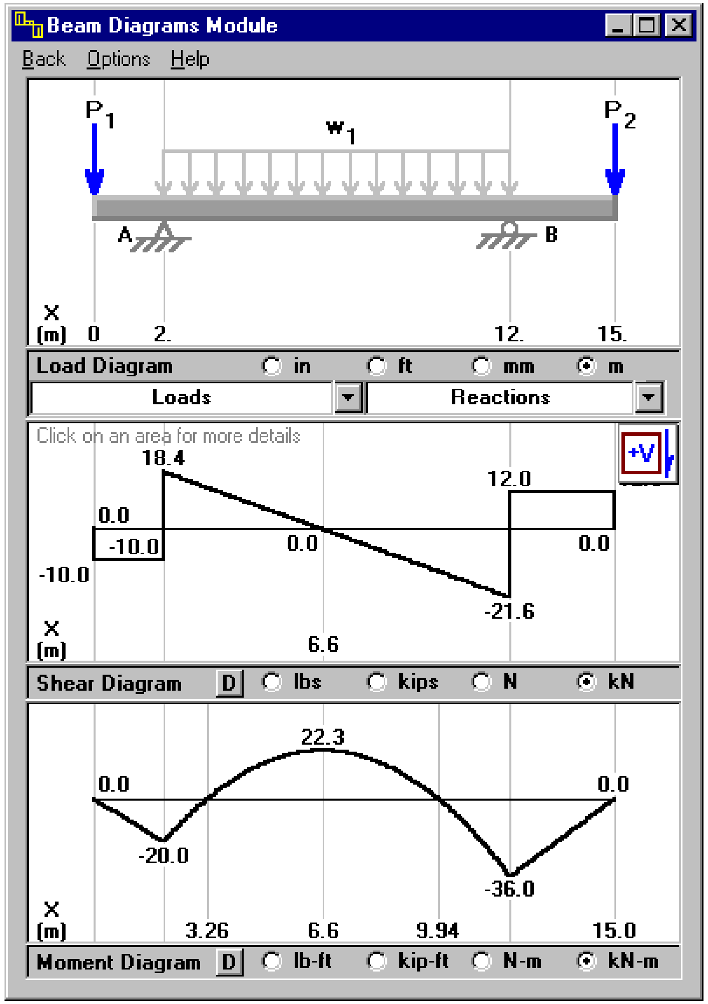
Transpose formulae

The world is not always linear !

$y = ax^n + bx^{n-1} \ldots\ldots + c$

Fluid mechanics, structural formulae, dynamics, etc are often non-linear

The world is not often linear !



Bending moments often in form

$M = ax^2 + bx + c$

http://www.mdsolids.com/screenshots/beamdiagram.htm

Solving equations with one variable

$ax^2 + bx + c = 0$

$ax^3 + bx^2 + cx + d = 0$

General approach

- get equation into the above forms
- solve either by (i) factor theorem, (ii) by algebra and/or (iii) graphically

Factor Theorem

Look at the non-x term, what factors could form the answer

e.g. $(x+d)(x+e) = ax^2 + bx + c = f(x)$

therefore d x e = c and when $x = -d$ or $-e$ $\quad f(x) = 0$

e.g. $x^2 - x - 6$

$x = 3$, $f(x) = 9-3-6 = 0$ therefore $(x - 3)$ is a factor

$6/-3 = -2$ therefore $(x + 2)$ is a factor

$f(x) = (x - 3)(x + 2) = 0$, $x = 3$ or -2

Factor Theorem

Cubic equations are more involved

e.g. $(x + e)(fx^2 + gx + h) = ax^3 + bx^2 + cx + d = f(x)$

$e \times h = d$ and when $x = -e$ $f(x) = 0$

$f = a$ $g + (e \times f) = b$ $h + (e \times g) = c$

x^3 term x^2 term x term

8 terms, four known quantities, guess 1

= three unknowns and three equations → can do

Cubic Equations – Long Division

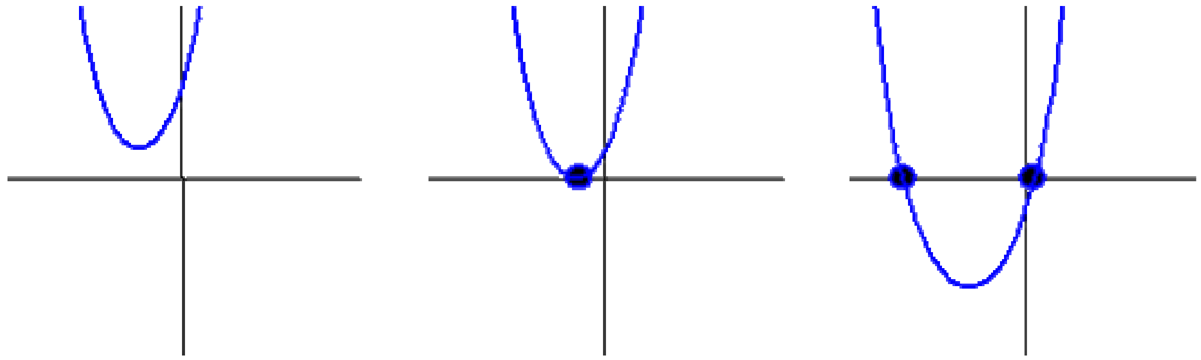
$x^3 + 4x^2 + 4x + 1 = 0 = y$

Guess a factor e.g. $x = -1$ $y = 0$, therefore $(x + 1)$ is a factor:

	$\underline{x^2 + 3x + 1}$	Final line formed
$x + 1$	$x^3 + 4x^2 + 4x + 1$	
Multiply by x^2	$\underline{x^3 + x^2}$	x^2 goes on top line
Subtract	$3x^2 + 4x + 1$	
Multiply by $3x$	$\underline{3x^2 + 3x}$	3x goes on the top line
Subtract	$x + 1$	
Multiply by 1	$\underline{x + 1}$	1 goes on the top line
	0	Complete

$0 = (x+1)(x^2 + 3x + 1)$, the final term can be solved by standard quadratic equation

Solutions to Quadratics

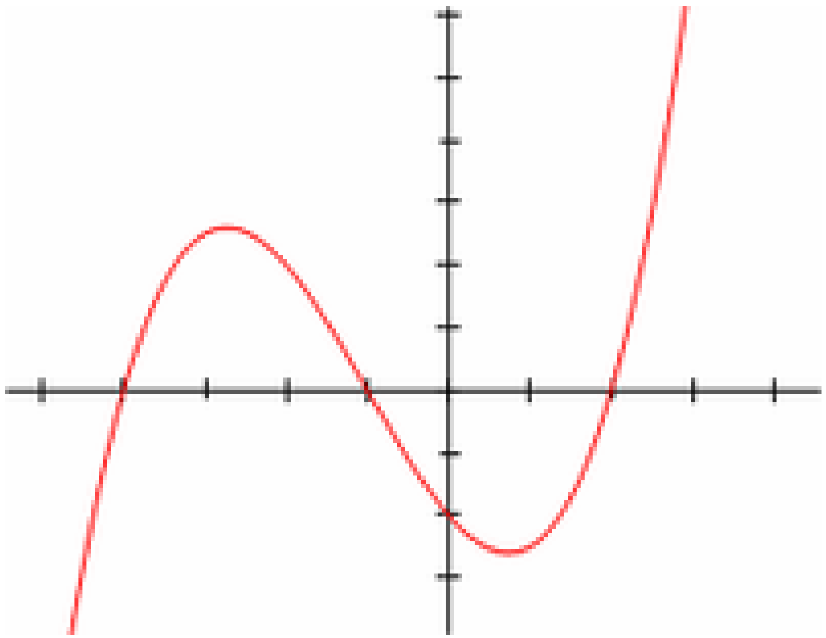


$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

http://www.jamesbrennan.org/algebra/quadratics/quadratic_definitions.htm

Cubic – Graphical Solution

$ax^3 + bx^2 + cx + d = 0$



Absolute Numbers

Modulus or absolute value function

$|x| = x$ for $x \geq 0$ and $-x$ for $x < 0$

Used in modification of wave signals in electronics

Wave Rectifiers

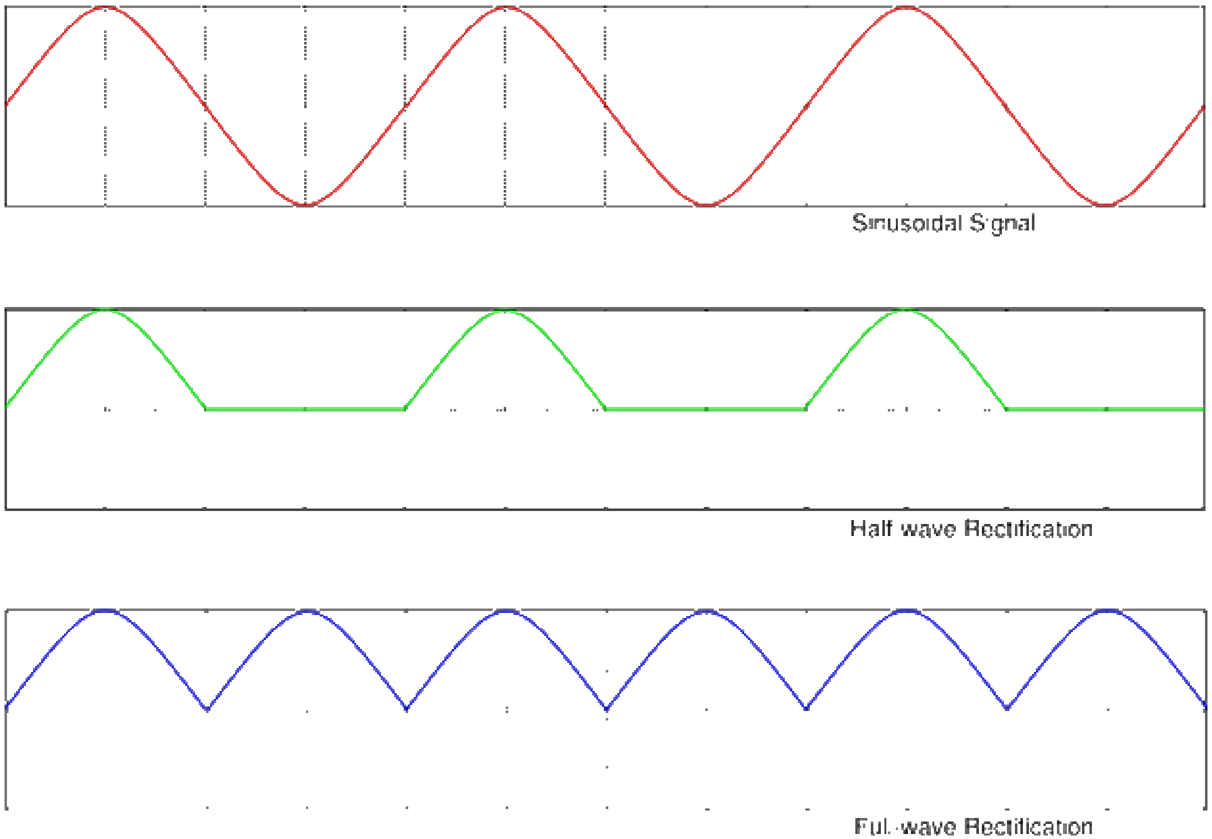


Image from www.wikipedia.org

Wave Rectifiers

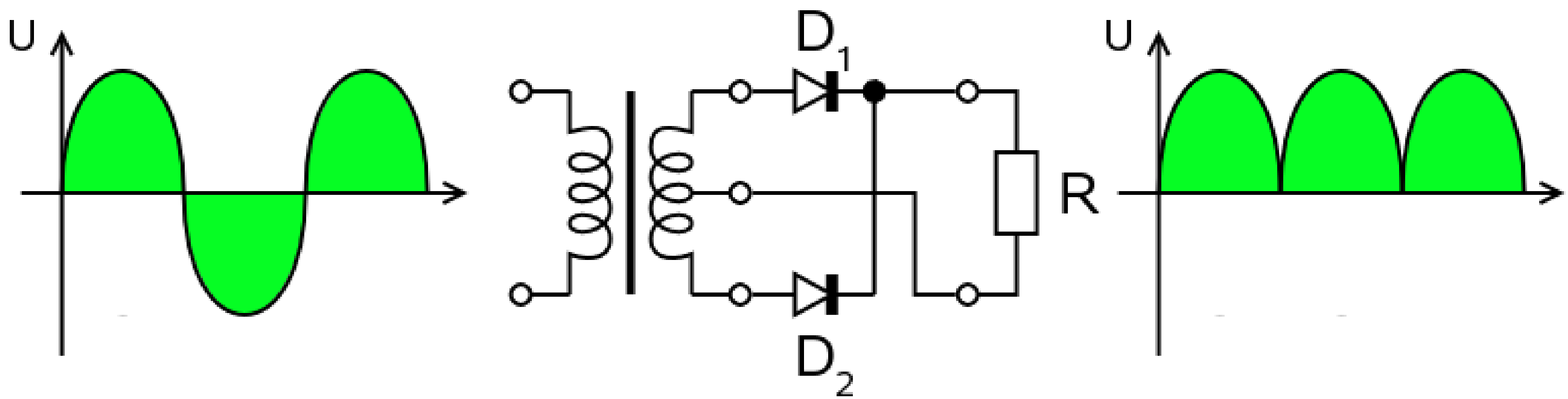


Image from www.wikipedia.org

Number Line

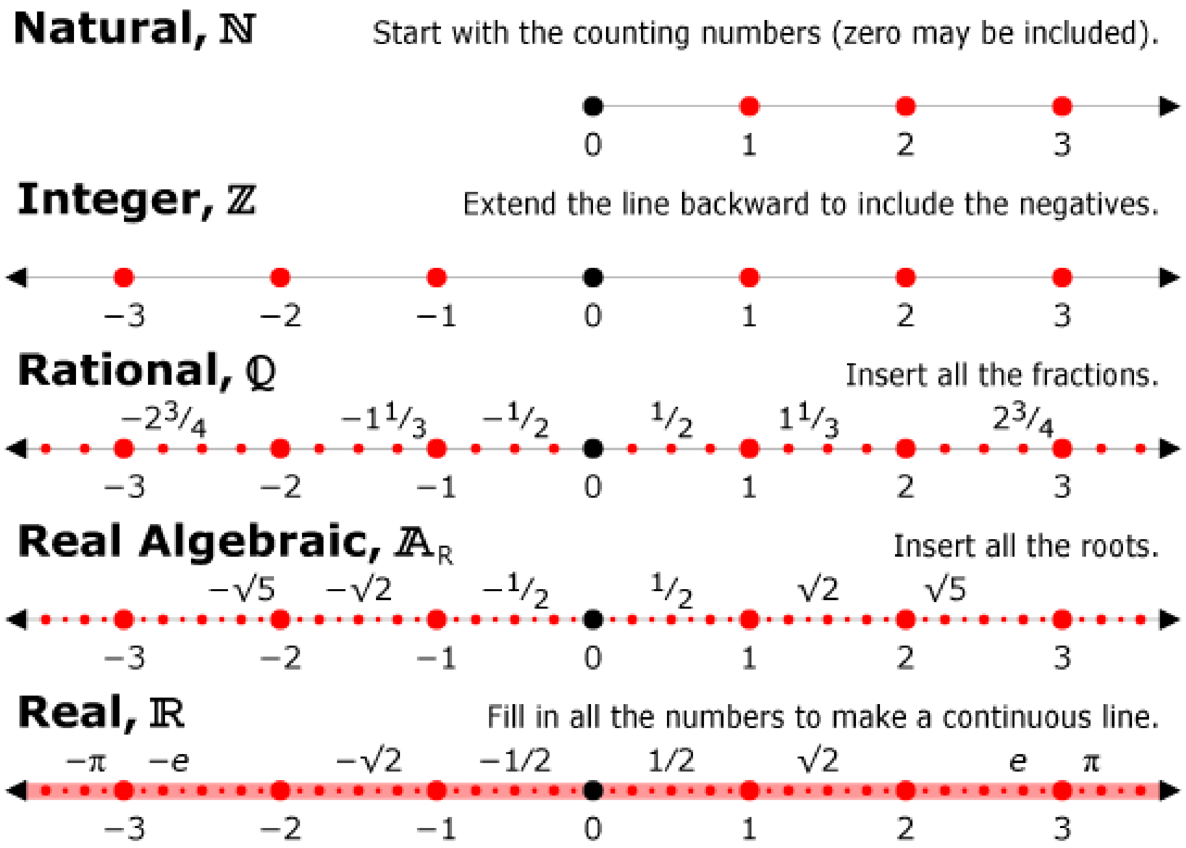


Image from thinkzone.wlonk.com/Numbers/NumberSets.htm

Real Numbers – Venn Diagram

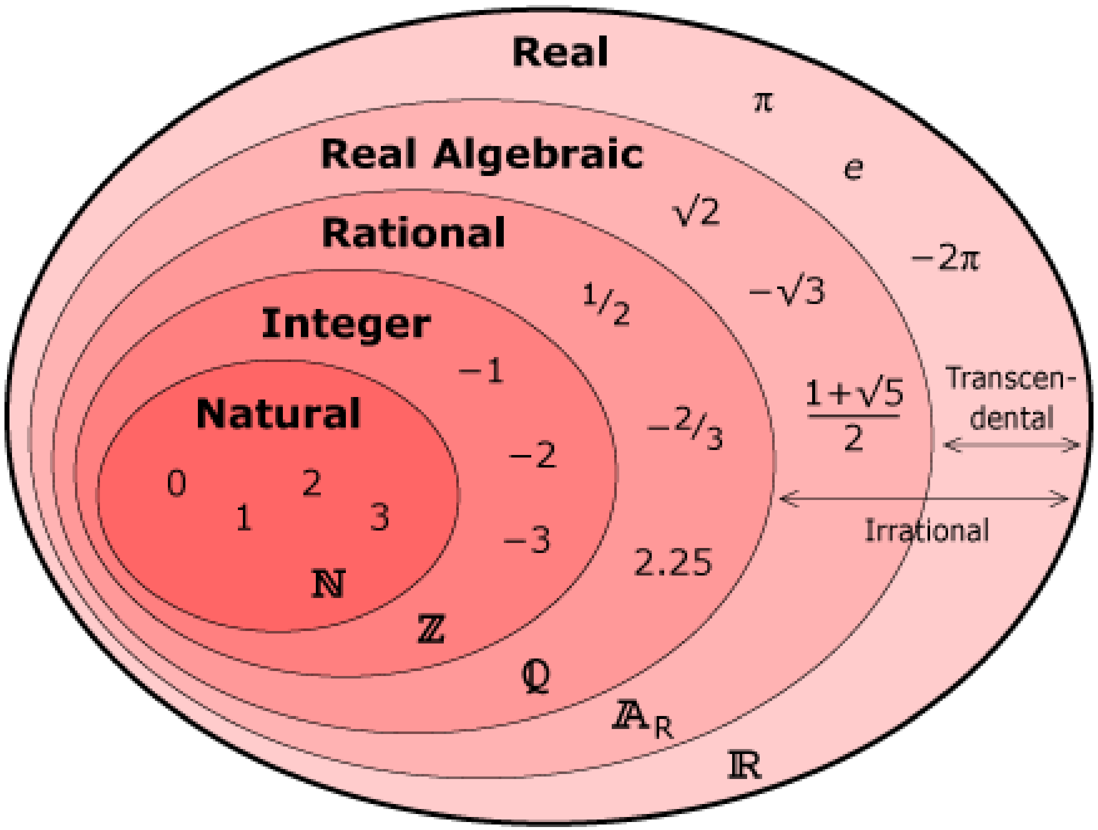
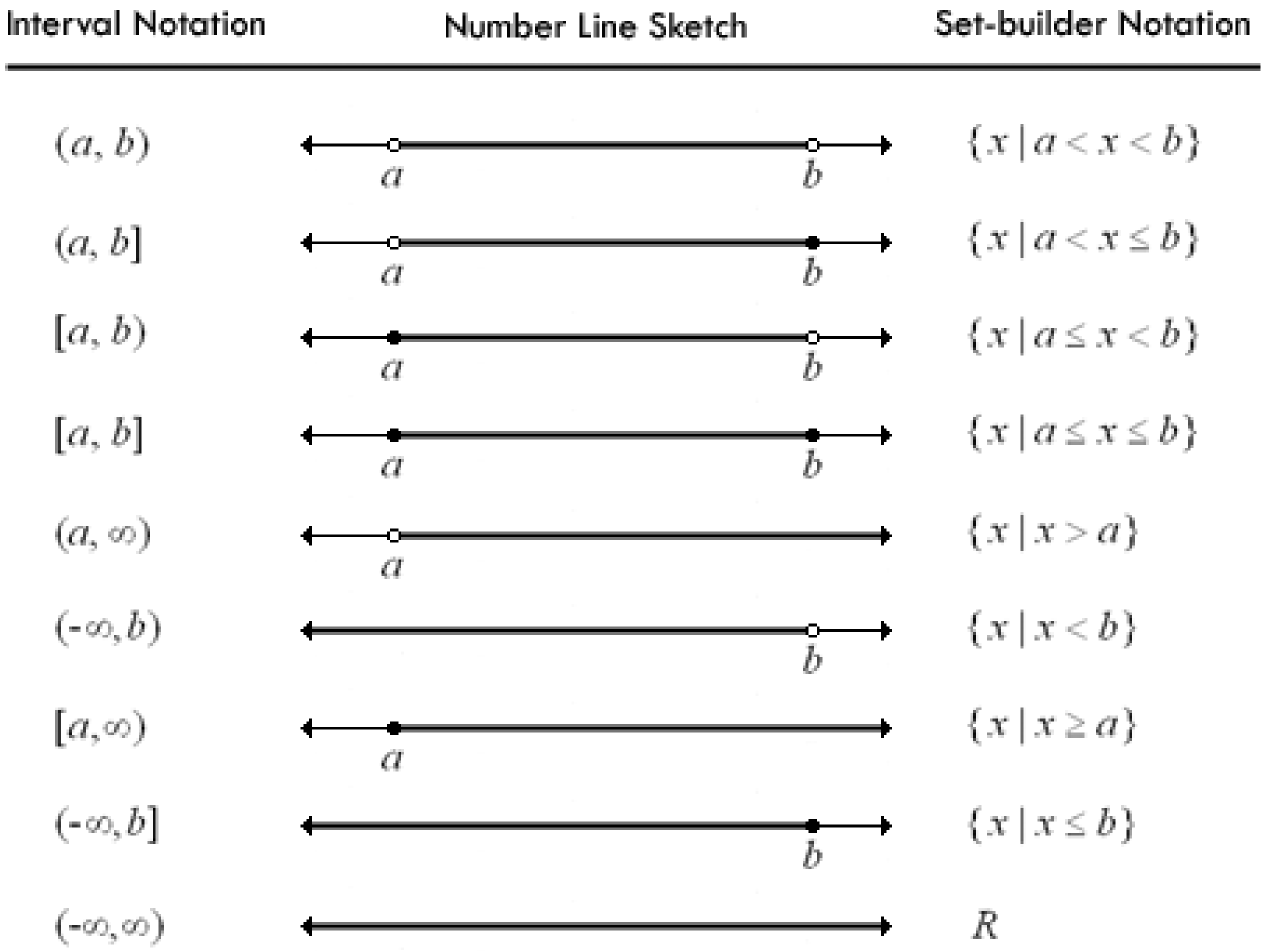


Image from thinkzone.wlonk.com/Numbers/NumberSets.htm

Open and Closed Intervals



Interval Notation	Number Line Sketch	Set-builder Notation
(a, b)	a b	$\{x \mid a < x < b\}$
$(a, b]$	a b	$\{x \mid a < x \le b\}$
$[a, b)$	a b	$\{x \mid a \le x < b\}$
$[a, b]$	a b	$\{x \mid a \le x \le b\}$
(a, ∞)	a	$\{x \mid x > a\}$
$(-\infty, b)$	b	$\{x \mid x < b\}$
$[a, \infty)$	a	$\{x \mid x \ge a\}$
$(-\infty, b]$	b	$\{x \mid x \le b\}$
$(-\infty, \infty)$		R

Image from http://www.nipissingu.ca/calculus/tutorials/sets.html

Evaluating Intervals

e.g. If $|2x - 1| < 5$ what is the range of x

Step 1 Expand absolute $-5 < 2x - 1 < 5$

Step 2 Simplify middle term $-5 + 1 < 2x < 5 + 1$

Step 3 Reduce to x $-2 < x < 3$ $(-2, 3)$

Step 4 Test solution by substitution

e.g. try $x = 4$ $|(2 \times 4) - 1|$ is not < 5

Inequalities – Graphical Solution

The result is illustrated in the following graph of $y = \frac{x-1}{2x+4}$ and $y = 1$, which shows that $\frac{x-1}{2x+4} < 1$ for $x < -5$ or for $x > -2$

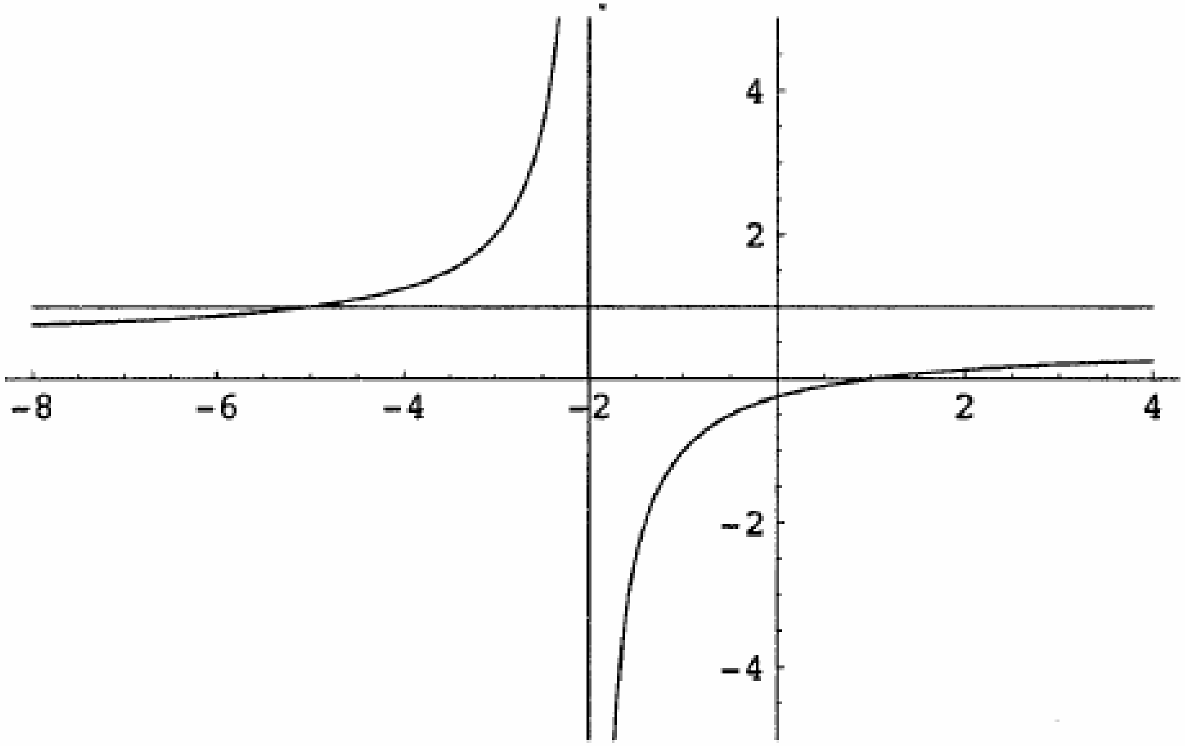


Figure 2: Graph of $y = \frac{x-1}{2x+4}$ and $y = 1$

Image from Barling et al. "Engineering Mathematics 1 Student Notes", 2009

Transposition of Formulae

- Show each step of the transposition clearly

  e.g. “multiply each side by sin x”

- Avoid miracles !

  e.g. $1/u = (1/x) + (1/y)$ does not mean $u = x + y$!!!!

- Check final formulae through i) units and/or ii) simple calculation

Transposition of Formulae

Example The solution of a differential equation is obtained in the form

$$-\frac{4}{y} = -\frac{1}{2x^2} + C$$

Re-arrange this equation to make y the subject.

Solution Multiply both sides of the equation by $-\frac{1}{4}$. Then

$$\frac{1}{y} = \frac{1}{8x^2} - \frac{C}{4}$$

Express the R.H.S. with a common denominator.

$$\frac{1}{y} = \frac{1 - 2Cx^2}{8x^2}$$

Take the reciprocal of both sides

$$y = \frac{8x^2}{1 - 2Cx^2}$$

Image from Barling et al. “Engineering Mathematics 1 Student Notes”, 2009

Transposition of Formulae

Example Given $Q = ka_1a_2\sqrt{\frac{2gH}{a_2^2 - a_1^2}}$, re-arrange the formula to make H the subject.

Solution Square both sides of the equation:

$$Q^2 = k^2a_1^2a_2^2\left(\frac{2gH}{a_2^2 - a_1^2}\right),$$

$$k^2a_1^2a_2^2\left(\frac{2gH}{a_2^2 - a_1^2}\right) = Q^2,$$

Multiply both sides by $\frac{a_2^2 - a_1^2}{k^2a_1^2a_2^2 2g}$ then

$$H = \frac{Q^2(a_2^2 - a_1^2)}{k^2a_1^2a_2^2 2g}.$$

Image from Barling et al. “Engineering Mathematics 1 Student Notes”, 2009