The Effect of Protuberance on Thermal Convection in a Square Enclosure

by

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Abstract

This paper documents research that has been undertaken at the Swinburne University of Technology in Melbourne Australia. In this paper, the analyses of heat transfer and free convective motion have been carried out numerically for a semi-cylindrically shaped protuberance in a square enclosure. The solution method is based on the finite element technique with the frontal solver. The numerical results are for a Prandtl Number 0.71 and for a Rayleigh Number up to 105. The change in direction of returning fluid near the cold wall effects the convective heat transfer process significantly. Moreover, increases in the protuberance affect the maximum velocities and their physical location in the domain resulting in dead zones near the bottom corners of the enclosures.

1. Introduction

Even after 40 years of research on the fluid flow and transport processes generated or altered by buoyancy forces in enclosures, many of the investigations found in the literature are related to regular shapes such as square, rectangular, cylindrical, annulus etc [1-5]. These results show that the conduction regime persists even at a Rayleigh Number as high as 500. In recent times many researchers have investigated the effect of curvature boundary on buoyancy driven flow. The curvature boundary can effect the transport of energy and momentum to an extent that the onset of convection establishes even at a very low Rayleigh Number ($\geq$10) [5-13]. Thus natural convection within a complex enclosure has gained considerable attention in recent times.

Despite the fact that the core flow within the enclosure is highly sensitive to the shape of its boundary and thermal boundary conditions, much of the work on natural convection is based on stream-vorticity formulation, where the vorticity or/and stream-function are not known explicitly. These are approximated through some kind of series expansion, which might lead to represent false boundary conditions and, in turn, induce an error. On the other hand, the primitive variables approach is more appropriate as the boundary conditions in terms of its velocity are known a priori.

In the present study, the effect of a semi-cylindrically shaped protuberance located at the lower surface of a long square cavity is studied numerically, similar to the work of Kaviany [13]. Kaviany used finite difference technique with stream-vorticity formulation and restricted his solution to a Rayleigh Number up to 10^4. In the present analysis, the authors used the finite element technique along with the frontal solver and velocity pressure formulation. The grid refinement with isoparametric elements allows for stable solutions at higher Rayleigh Numbers than those of Kaviany [13], up to $Ra=10^5$.

2. Problem Definition and Governing Equations

A two-dimensional steady-state analysis of an incompressible fluid, driven by the buoyancy force is examined in a square enclosure with a protuberance in the shape of semicircle located symmetrically at the bottom surface. The steady-state governing equations are sought. Using the Boussinesq approximation and neglecting the dissipation effect due to the viscous term and no heat generation, the governing equations in non-dimensional form are written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\Pr\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right] \qquad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\Pr\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]+T.Ra.\Pr \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

Equations (1-4) were normalized using the following dimensionless scales:

$(x,y) = (x^*,y^*)/L_{ref}$; $(u,v) = (u^*,v^*)L_{ref}/\alpha$; $p = p^*L^2_{ref} / (\rho\alpha^2)$;

$T = (T^* - T_c)/(T_h - T_c)$; $Ra = g\beta(T_h - T_c)L^3_{ref} / (\nu\alpha)$; $Pr = \nu/\alpha$;

Where asterisks denote dimensional variables.

In obtaining the numerical solution of for the enclosure, the following boundary conditions are used.

At x = 0,   u = v =0 & T_h=1; At x = 1, u = v =0 & T_c=0;

At y = 0,   u = v =0 & $\frac{\partial T}{\partial n} = 0$; At y = 1, u = v =0 & $\frac{\partial T}{\partial n} = 0$

3. Finite Element Formulation

The velocity and thermal energy equations (1-4) result in a set of non-linear coupled equations for which an iterative scheme is adopted. The total domain is discretised into 1202 elements that results in 3799 nodes. All the elements are isoparametric, quadrilateral containing eight nodes. All eight nodes are associated with velocities as well as temperature; only the corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and which is satisfied through continuity equation.

By employing the Galerkin weighted residual approach and substituting appropriate shape functions, the governing equs. (1-4) can be written as;

Conservation of mass:

$$\sum_{1}^{n^e} \int_{A^e} M_l \left[\frac{\partial N_j}{\partial x} u_j + \frac{\partial N_j}{\partial y} v_j \right] dA^e = 0 \tag{5}$$

$$\sum_{1}^{n^e} \int_{A^e} \Bigg[N_i N_k u_k \frac{\partial N_j}{\partial x} u_j + N_i N_k v_k \frac{\partial N_j}{\partial y} u_j + N_i \frac{\partial M_l}{\partial x} p_l + \\ \Pr \left[\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} u_j + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} u_j \right] \Bigg] dA^e = \int_{\Gamma^e} \Pr N_i \frac{\partial u}{\partial n} d\Gamma^e \tag{6}$$

$$\sum_{1}^{n^e} \int_{A^e} \Bigg[N_i N_k u_k \frac{\partial N_j}{\partial x} v_j + N_i N_k v_k \frac{\partial N_j}{\partial y} v_j + N_i \frac{\partial M_l}{\partial y} p_l + \\ \Pr \left[\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} v_j + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} v_j \right] - Ra \Pr N_j T_j \Bigg] dA^e = \int_{\Gamma^e} \Pr N_i \frac{\partial v}{\partial n} d\Gamma^e \tag{7}$$

$$\sum_{1}^{n^e}\int_{A^e}\left[N_i N_k u_k \frac{\partial N_j}{\partial x}T_j + N_i N_k v_k \frac{\partial N_j}{\partial y}T_j + \left[\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x}T_j + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}T_j\right]\right]dA^e$$

$$= -\int_{\Gamma^e} N_i \frac{\partial T}{\partial n} d\Gamma^e \tag{8}$$

The outer summation refers to each element in the domain and the integration is valid over the area of an element (A^e). The boundary of the element (Γ^e) are summated in the adjacent element and the net contribution becomes zero unless the boundary which acts as a limit to the domain is encountered. The above Equs. can be combined to form a assembled matrix equation as;

[A][X] = [B]

where the chosen form for X is,

$$X = \begin{bmatrix} u_j \\ p_l \\ v_j \\ T_j \end{bmatrix} \tag{9}$$

And each coefficient in matrix A has the form,

$$a_{ij} = \sum_{1}^{n^e}\int_{A^e}\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} dA^e$$

where

$$c_{11} = \left[N_i\ N_k u_k \frac{\partial N_j}{\partial x} + N_i\ N_k v_k \frac{\partial N_j}{\partial y}\right] + \Pr\left[\frac{\partial N_i}{\partial x}\frac{\partial Nj}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial Nj}{\partial y}\right];$$

$c_{12} = N_i \dfrac{\partial M_1}{\partial x}$; $\quad c_{13} = 0$; $\quad c_{14} = 0$;

$c_{21} = M_1 \dfrac{\partial N_j}{\partial x}$; $\quad c_{22} = 0$; $\quad c_{23} = M_1 \dfrac{\partial N_j}{\partial y}$;

$c_{24} = 0$; $\quad c_{31} = 0$; $\quad c_{32} = N_i \dfrac{\partial M_1}{\partial y}$;

$c_{33} = c_{11}$; $\quad c_{34} = -Ra\Pr N_j$; $\quad c_{41} = 0$;

$c_{42} = 0$; $\quad c_{43} = 0$;

$$c_{44} = N_i\ N_k u_k \frac{\partial N_j}{\partial x} + N_i\ N_k v_k \frac{\partial N_j}{\partial y} + \left[\frac{\partial N_i}{\partial x}\frac{\partial Nj}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial Nj}{\partial y}\right];$$

and each coefficient of matrix B has the form

$$b_i = \sum_{1}^{n^e} \int_{\Gamma} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} d\Gamma^e \qquad (10)$$

where

$b_1 = \Pr N_i \dfrac{\partial u}{\partial n}$; $\quad b_2 = 0$; $\quad b_3 = \Pr N_i \dfrac{\partial v}{\partial n}$; $\quad b_4 = -N_i \dfrac{\partial T}{\partial n}$;

4. Numerical Solution

There is a standard procedure for evaluating the above integrals, which is described in a number of textbooks and papers in the field of finite element analysis. [12] This involves (i) the normalisation of the coordinates, and (ii) the use of Gauss-Legendre quadrature scheme. In the present work, 3x3 and 3x1 Gaussian integration sampling point schemes are used for the surface and line integral respectively. The resultant nonlinear nonsymmetric matrix equations are solved by the frontal algorithm for which a suitable element numbering scheme is also adopted in order to reduce the front- width.

5. Results and Discussions

5.1 Overview

Numerical results were obtained for wide range of Rayleigh Numbers ($10 \le Ra \le 10^5$) and protuberance diameter ranges from 0.2 to 1 and Pr=0.71. The circular section of the protuberance was divided into 32 elements resulting 64 number of nodes. Moreover, the grid spacings near the protuberance and enclosure corners were very small to capture the existence of any flow reversal phenomena.

5.2 Temperature and velocity Fields

The isotherms for Ra $=10^2$, 10^3, 10^4, and 10^5 were measured for the case of bottom surface of protuberance with D=0.5. The low Rayleigh number was characterised by a parallel flow structure. In the parallel flow region, conduction is the dominant mode of heat transfer. However, in the lower bottom corner there bound to be some amount of convection even at a very low Rayleigh number. As Ra increases, a stronger convective motion establishes. Consequently, near both hot and cold walls, the isotherms become closely spaced with thermal stratification in the core region, indicating higher heat transfer rate. However, even at a high Rayleigh number (Ra=10^5), the bottom corners still showed vertical isothermal lines, a quasi conduction region, indicating negligible influence of core flow in these corners. Isotherm patterns for Rayleigh number 10^5 at different diameters of protuberance showed that thermal stratification decreased as the diameter of protuberance increased and hence there was a decrease in the convective current. Thus the decrease in convective current is mainly attributed to the protuberance at the bottom of the enclosure. Kaviany[13] has described this as a no-slip boundary condition on the protuberance which tends to suppress the formation of vortices.

The cold fluid near the cold wall, which descended with the greater velocity due to a gravity force was subjected to a larger change in direction as the diameter of protuberance increases. With the increase in diameter and at higher Rayleigh number, this fluid layer parts from the bottom cylindrical surface and transforms into a plume flowing upward and mixes with the entire volume of fluid. This inturn effect the down-word flow near the cold wall and the whole convective process. Thus with the aim of analysing the effects of protuberance due to the increase in its diameter, the authors focused on the velocity patterns more intensively at increasing protuberance diameter for wide range of Rayleigh numbers.

The +ve u-velocity at the top and –ve u velocity at the bottom of the enclosure indicated a clockwise unicellular rotation at the central part of the domain. At $Ra \le 10^4$, the max u-velocities lies exactly in the mid-vertical axis for all range of protuberances. However, the max v-velocities occurs slightly above the mid-horizontal line and get shifted to further up with the increase of diameter of protuberance. Further increase in Rayleigh number ($Ra=10^5$) the maximum u-velocities migrates towards the top hot wall and bottom cold wall corner, and these observation are very much like those of a square enclosures. However, the maximum v-velocities shift towards the top cold and bottom hot wall corners, which is the main cause of distortion in the core flow.

The u velocity in the mid vertical axis was also examined. Although, the velocity pattern indicated the symmetric nature of u velocity in the range of $Ra \le 10^{4,}$ the domain loses its symmetry even before the onset of convection and becomes more significant as the Rayleigh Number increases. This effect becomes more pronounced at higher protuberance diameters. It was also observed from the u-v contour plots that the velocities in the two bottom corners were approximately 2% of the maximum velocity and decreased further with increases in diameter of protuberance. At D=1.0 these corners behaved completely as dead zones with no circulations at all.

The measured indicated that the u-velocity near the top horizontal wall was higher than u-velocity in the bottom wall and v-velocity near the hot vertical wall always dominated over the v-velocity near the cold wall for all range of Rayleigh Numbers and protuberances. However, it was expected that the cold returning fluid should have higher velocity as the buoyancy force supports it. It does not happen so, because the cold returning fluid is subjected to sudden change of direction due to protuberance at the bottom and this is even more when the diameter of protuberance increases. With increase in radius of protuberance these velocities shift more toward to their respective walls. At lower range Rayleigh number maximum v-velocities were almost same neat hot and cold walls. However, at a higher Rayleigh Number ($Ra>10^4$)

v-velocity near the hot wall was higher than the downward v-velocity near the cold wall.

5.3 Nusselt Number Calculation

The Nusselt Number for the isothermal wall of the enclosure is derived from the energy balance equation as follows:

$$\overline{Nu} = \frac{hL_{ref}}{k} = -\frac{\partial T}{\partial n} \tag{11}$$

The average Nusselt Number over the total length of the hot wall is:

$$\overline{Nu} = \frac{1}{L_{ref}} \int_0^L -\frac{\partial T}{\partial n} dn \tag{12}$$

The average Nusselt Number variation with Rayleigh number was studied. The Nusselt Number increased monotonically with increase in Rayleigh number. When different radii of protuberance were compared it was found that the protuberance had a significant effect on the heat transfer effect. The rate of heat transfer decreased with the diameter and it was more pronounced at higher Rayleigh number. At higher diameter of protuberance and higher Rayleigh number the velocities near the hot and cold wall decreased significantly and established a poor convective current throughout the domain. A flat or a concave bottom adiabatic bottom would be more advantageous in terms of higher heat transfer rate because it would allow a smooth passage to the returning fluid from the cold wall and less diffusion towards the central part of the enclosure, which would establish better convective current.

6. Conclusion

The present numerical scheme was validated for accuracy in the prediction of buoyancy driven flow within enclosures having curvature boundary. The predicted results showed that protuberance at the bottom of the enclosure had a significant effect on the convective flow and heat transfer characteristics. At a higher Rayleigh Number, the protuberance affected the entire convective flow phenomena within the domain as it allowed the flow to mix with core flow. However, a flat and/or concave adiabatic bottom would be advantageous to establish better convective current within the enclosure.

7. Nomenclature:

E	Offset (height of dome from top of the isothermal wall)
e	Eccentricity
g	Acceleration due to gravity, m/sec^2
H	Height of the enclosure, m
h	Coefficient of Heat Transfer, W/m^2 K
L_{ref}	Characteristic length, m
n	Outward flux normal to boundary
$\overline{Nu}$	Average Nusselt number
p	Dimensionless fluid pressure
Pr	Prandtl number
Ra	Rayleigh number
T	Dimensionless fluid temperature,
u,v	Dimensionless velocity components in x and y directions respectively.
x,y	Dimensionless coordinates
Greek symbols	
α	fluid thermal diffusivity
β	Coefficient of volumetric expansion, $^oC^{-1}$
ν	Fluid kinematic viscosity, m^2/sec
ρ	Fluid density, kg/m^3
γ	Aspect ratio
Subscript	
h,c	Hot and cold wall respectively

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