New measurements and new physics with spinor BEC

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Optical trapping

First suggested by Lekhotov (1978)
Demonstrated by Chu (1986)
Applied to BEC (Stamper-Kurn 1998)
All-optical BEC (Chapman 2001)

Some initial experiments on spinor BEC: domain formation and interactions (MIT group 1998-9)

Feshbach resonances
Tunable interactions
Molecule formation
Degenerate Fermi gases
BEC/BCS crossover
What is a spinor BEC?

Magnetic traps hold only one sign of spin projection.

\[ \psi(r) = \sqrt{n(r)} e^{i\theta(r)} \]

Optical traps hold all spin projections

\[ \psi(r) = \sqrt{n(r)} \begin{pmatrix} \xi_{-F}(r) \\ \vdots \\ \xi_{+F}(r) \end{pmatrix} \]

\[ \xi^T(r)\xi(r) = 1 \]
Stern-Gerlach imaging

Put the BEC in a magnetic gradient
Spin components separate
Image with absorption imaging

Barrett, Sauer, Chapman
PRL 87 010404 (2001)
Symmetry and interactions

Interactions

For experimentalists

Contact potential but with $F+1$ scattering lengths

$$V(\mathbf{r}_1 - \mathbf{r}_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2) \frac{4\pi\hbar^2}{m} \sum_{f=0,2,...}^{2F} a_f P_f$$

Odd $f$ forbidden by Bose symmetry.

MDDI small but measurable for $F=1$

For theorists

Projection onto total spin
Spin-mixing oscillations

$^{23}\text{Na } c > 0$

NIST

Black, Gomez, LDT, Jung, Lett
PRL 99 070403 (2007)

$^{87}\text{Rb } c < 0$

GATech

Nat Phys 1 111 (2005)
F=1 interactions

Only one spin-changing interaction allowed is

\[ |+1> \leftrightarrow |-1> \leftrightarrow |0> |0> \]

\[ V(r_1 - r_2) = \delta(r_1 - r_2)(c_0 + c_2 F_1 \cdot F_2) \]

Order parameter has three components

\[ \psi(r, t) = \sqrt{n(r)} \begin{pmatrix} \xi_{-1}(r, t) \\ \xi_0(r, t) \\ \xi_{+1}(r, t) \end{pmatrix} \]

Three coupled Gross-Pitaevski equations:

\[ i\hbar \frac{\partial \psi_1}{\partial t} = L_1 \psi_1 + c_2 (n_1 + n_0 - n_{-1}) \psi_1 + c_2 \psi_0^2 \psi_{-1}^*, \]

\[ i\hbar \frac{\partial \psi_0}{\partial t} = L_0 \psi_0 + c_2 (n_1 + n_{-1}) \psi_0 + 2c_2 \psi_0^* \psi_1 \psi_{-1}, \]

\[ i\hbar \frac{\partial \psi_{-1}}{\partial t} = L_{-1} \psi_{-1} + c_2 (n_{-1} + n_0 - n_1) \psi_{-1} + c_2 \psi_0^2 \psi_{+1}^*, \]
Topological defects galore

- Spin textures
- Spin vortex lattices
- Coreless vortices
- Nematic disclinations
- Spin knots
- Spin solitons
Single mode approximation

Assume that the spatial wavefunction is:

- constant
- identical for all three spin components

\[
\psi(r, t) = \sqrt{n(r)} \left( \begin{array}{c}
\xi_{-1}(t) \\
\xi_0(t) \\
\xi_{+1}(t)
\end{array} \right)
\]

Further break up into populations \( \rho \) and phases \( \theta \)

\[
\psi(r, t) = \sqrt{n(r)} \left( \begin{array}{c}
\sqrt{\rho_{-1}(t)} e^{i\theta_{-1}(t)} \\
\sqrt{\rho_0(t)} e^{i\theta_0(t)} \\
\sqrt{\rho_{+1}(t)} e^{i\theta_{+1}(t)}
\end{array} \right)
\]
Magnetisation and the linear Zeeman effect

**Magnetisation** is \( m = \rho_+ - \rho_- \)

Constant of the motion

Populations normalised

\[
\rho_+ + \rho_0 + \rho_- = 1
\]

Whole system described by:

One population, say \( \rho_0(t) \)

Overall phase \( \theta = \theta_+ + \theta_- - 2\theta_0 \)

\[
E = c\rho_0 \left( 1 - \rho_0 + \sqrt{(1 - \rho_0)^2 - m^2 \cos \theta} \right) + \delta(1 - \rho_0)
\]

Spin interaction

\( c > 0 \) for antiferromagnetic \(^{23}\text{Na}\)
Effective Hamiltonian for F=1 in SMA

\[ E = c \rho_0 \left( 1 - \rho_0^2 + \sqrt{(1 - \rho_0^2)^2 - m^2 \cos \theta} \right) + \delta (1 - \rho_0^2) \]

Spin interaction

Quadratic Zeeman
Destructive Stern-Gerlach measurements

![Graph showing magnetic field in μT on the x-axis and oscillation period in ms on the y-axis. The graph exhibits a peak and a spread around the data points.]
Faraday measurement of transverse magnetisation

Spin precesses around bias field

Polarization alternately rotated left and right.

No rotation when spin is perpendicular to beam.

Put cube at 45° to input polarization, get bright field signal that is nulled for no rotation.

Net effect is a signal oscillating at the Larmor frequency.
Faraday measurement, details

Experimentalist here

Theorists here

Calibrated aperture on xy stage

Image of BEC ~0.5 mm

2 mm

20 μm

Compound lens:
1.8x relay, achromatic pair
11x telescope
20x total magnification

\[ \theta_{\text{rot}} \propto \int F_x \, dx \]
Faraday measurement: spin oscillations

Magnetic field = 26 uT

Spectral power density
Spin oscillations: period divergence

Magnetization = 0
Decoherence in the F=1 SMA system
Single-mode ground state
Non-linear Landau-Zener tunneling

\[ H(\gamma) = \begin{pmatrix} \frac{\gamma}{2} & -\frac{V}{2} \\ -\frac{V}{2} & -\frac{\gamma}{2} \end{pmatrix} \]

Lucas Rutten, Monash
Summary: single-mode spin-1 system

We now understand:

- Free evolution
- Ground state ...
- ... and how it gets there (decoherence)
- Adiabatic(ish) evolution
Anything else?

Shapiro steps

Phenomenology in Josephson systems
Add a magnetic field dither
Macroscopic Quantum State Trapping (MQST)

Spin echoes of magnetic dipole-dipole interaction (MDDI)

FIG. 3. Microwave power at 9000 Mc/sec (A) and 24000 Mc/sec (B) produces many zero-slope regions spaced at $h v / 2 e$ or $hv / e$. For $A$, $hv / e = 38.5 \mu$W, and for $B$, 103.

\[ \omega_1 t = \begin{align*}
\text{(a)} & : 0.75 \\
\text{(b)} & : 2.4 \\
\text{(c)} & : 4.1 \\
\text{(d)} & : 5.8 \\
\text{(e)} & : 7.3 \\
\text{(f)} & : 9.1 \\
\text{(g)} & : 10.6 \\
\text{(h)} & : 12.8
\end{align*} \]
Beyond mean-field: Spinor squeezing

\[ \mathcal{E} = c\rho_0[(1 - \rho_0) + \sqrt{(1 - \rho_0^2) - m^2 \cos \theta}] + \delta(1 - \rho_0). \]

\[ \mathcal{H}_{\text{eff}} = \frac{c}{2} \left[ \hat{\rho}_0 \sqrt{(1 - \hat{\rho}_0^2) - m^2 \cos \hat{\theta}} + \cos \hat{\theta} \hat{\rho}_0 \sqrt{(1 - \hat{\rho}_0^2) - m^2} \right] + (1 - \hat{\rho}_0)(c\hat{\rho}_0 + \delta). \]

Chang et al, PRL 99 080402 (2007)
Beyond single mode

$^{23}\text{Na}$ Antiferromagnetic has lower energy

Won't form domains:
Robins et al. PRA 64 021601R (2001)
Zhang et al. PRL 95 180403 (2005)

Can form domains:
Alexander et al. PRA 78 023632 (2008)

$^{87}\text{Rb}$ Ferromagnetic has lower energy

(Stern-Gerlach)
Application: micromagnetometry
Conclusions

Optical trapping $\Rightarrow$ spinor order parameter

Single-mode $F=1$ model

Faraday measurement
- Well adapted to measuring a single spatial mode
- “NMR” extensions, spinor squeezing ...

Microscale atomic magnetometers