Optical cavity QED

Luis A. Orozco
Joint Quantum Institute
Department of Physics
Lecture 2
Typical system optical experiments.

\[ \frac{g}{2\pi} = 6 \text{ MHz} \]

\[ \frac{\kappa}{2\pi} = 3.6 \times 10^6 \text{ s}^{-1} \]

\[ \frac{\gamma}{2\pi} = 6.0 \times 10^6 \text{ s}^{-1} \]

\[ C_1 = \frac{g^2}{\kappa \gamma} \approx 2 \]

\[ n_0 = \frac{\gamma^2}{3g^2} \approx 0.1 \]

Expansion parameters \( 1/n_0 \) and \( C_1 \)
We want to study Quantum Phenomena in the TIME DOMAIN: Dynamics
We looked at the frequency and time response to step excitation.

We want now to go to a situation where there is no explicit time dependence in the excitation. This is different from the step excitation.
What is strong coupling in cavity QED.

The quantum fluctuations are comparable greater than the mean. The size of the fluctuations is set by $C_1$ and by $1/n_0$.

Possibility to study an open quantum system.
The study of noisy signals is done with correlation functions. We have learned a lot about characteristic times, sizes and some dynamics with such correlation functions in statistical mechanics. They have the following form:

\[ \langle F(t) F(t+\tau) \rangle \]
\[ \langle F(t) G(t+\tau) \rangle \]

For Optical signals the variables we want to correlate to themselves or to each other can be:

Field and Intensity
How do we measure these functions?

\[ G^{(1)}(t+\tau) = <E(t) E^*(t+\tau)> \quad \text{field-field} \]
\[ G^{(2)}(t+\tau) = <I(t) I(t+\tau)> \quad \text{intensity-intensity} \]
\[ H(t+\tau) = <I(t) E(t+\tau)> \quad \text{intensity-field} \]
Wave-Wave Correlation
Michelson Interferometer

\[ g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau) \rangle}{\langle I(t) \rangle} \]

Spectrum of the source

\[ F(\omega) = \frac{1}{2\pi} \int exp(i\omega\tau)g^{(1)}(\tau)d\tau \]

Basis of Fourier Transform Spectroscopy
Classical Intensity Correlation Functions: Hanbury Brown and Twiss. (HTB) two persons only!

\[ g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t) \rangle^2} \]

At equal time:

\[ g^{(2)}(0) = \frac{\langle I^2 \rangle}{\langle I \rangle^2} = \frac{\langle (\Delta I)^2 \rangle}{\langle I \rangle^2} + 1. \]

With the variance:

\[ \langle (\Delta I)^2 \rangle = \langle I^2 \rangle - \langle I \rangle^2 \]

The equal time correlation function is related to the variance, and will be greater than 1.
How do we measure such correlations:

Construct the “Periodogram”

Take the photocurrent which is proportional to the intensity $I(t)$

$$I(t) \rightarrow I_i$$
$$I(t + \tau) \rightarrow I_j$$

$$\langle I(t)I(t + \tau) \rangle \rightarrow \sum_{i=0}^{M} \sum_{n=0}^{N} I_i I_{i+n}$$

Discretize (digitize) the time series. Calculate the correlation by displacing the series by a fixed number $n$, multiply and then sum and average. Careful to normalize each sum in a way to reveal that the size of the sample may be finite.
Discretize the time series:

\[ I_i \]

Multiply by the delayed time series

\[ I_{i+n} \]

Average the result and see the “correlation time”
Another way to calculate the correlation function is with the waiting time distribution.

Measure the separation between two consecutive pulses (start and stop)

Histogram the distribution of separations.

This is the same as $g^{(2)}(\tau)$ if the fluctuations are very rare.

Make sure the intensities are low, so that seldom you get coincidences.
Example of how to measure $g^{(2)}(\tau)$ with the time series and the waiting time distribution.

Digital storage oscilloscope (DO) captures the photocurrent out of the Photomultiplier tube (PMT) for a long time and then process the time series.

Photon correlator with Avalanche Photo-Diodes (APD), waiting time distribution. The Time to Digital Converter (TDC) can register up to 16 stops.
Auto- and cross-correlation with a digital storage oscilloscope

Average TTL pulses on oscilloscope as an analog signal
Comparison of $g^{(2)}(\tau)$ from photon counting (a), and from a time series of intensities measured with a single PMT (b).
Correlation functions in quantum optics are conditional measurements.

The detection of the first photon prepares a state that then evolves in time.

The correlation functions can be Field-Field: Mach-Zehnder, Interferogram.
Intensity-Intensity: $g^{(2)}(t)$ Hanbury-Brown and Twiss.

When the dynamics of the conditioned state are slow enough it is possible to feedback to the system and modify its state.
Very brief and incomplete history of the intensity correlation functions.

1956 Hanbury-Brown and Twiss; astronomy to measure the size of a star looking at the intensity with two different detectors, not interfering the fields as Michelson had proposed and done. This is a classical effect of bunching.

1963 Glauber and others (Mandel, Sudarshan, Wolf) formalize the quantum correlation functions “Quantum theory of Coherence”.

1976 Experiment and Theory of single atom resonance fluorescence Kimble, Dagenais, Mandel; Carmichael and Walls.

Shows the non-classical effect of antibunching.

Earlier experiments for Bell Inequalities by Clauser with a cascaded atomic source can be interpreted as well as measurements of non-classical properties of light.
The variance of the number of photons is related to the probability of coincident photons given by $g^{(2)}(0)$. Light with Poissonian statistics has $g^{(2)}(0) = 1$. Light with a super-Poissonian statistical distribution has $g^{(2)}(0) > 1$. A sub-Poissonian distribution has $g^{(2)}(0) < 1$, a clear signature of a nonclassical field.

Quantum Mechanically (time and normal order):

$$ g^{(2)}(\tau) = \frac{\langle T : \hat{I}(t) \hat{I}(t + \tau) : \rangle}{\langle \hat{I}(t) \rangle^2} $$

$$ g^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2}. $$

$$ \sigma^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \quad g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}. $$
Intensity correlation function measurements:

\[ g^{(2)}(\tau) = \frac{\langle \hat{I}(t)\hat{I}(t+\tau) \rangle}{\langle \hat{I}(t) \rangle^2} \]

Gives the probability of detecting a photon at time \( t + \tau \) given that one was detected at time \( t \). This is a conditional measurement:

\[ g^{(2)}(\tau) = \frac{\langle \hat{I}(\tau) \rangle_c}{\langle \hat{I} \rangle} \]
Experimental considerations

Collimated atomic beam: thermal, high velocity only a few atoms are maximally coupled.

Atomic number fluctuations are small.
mean = 25

The graph shows the function $g^{(2)}(\tau)$ as a function of $\tau$ (ns). The data points are scattered around the mean value of 25, with a horizontal line indicating the reference value of 1.0.
$g^{(2)}(\tau)$

$\tau$ (ns)

mean = 37
mean = 62

$g^{(2)}(\tau)$

$\tau$ (ns)
mean = 75

$g^{(2)}(\tau)$

$\tau$ (ns)
mean = 87
mean = 186

$g^{(2)}(\tau)$ vs $\tau$ (ns)
mean = 262
mean = 362
mean = 548
mean = 792

\[ g^{(2)}(\tau) \]

\( \tau \text{ (ns)} \)
The state of the cavity QED system for N atoms is:

\[ |\psi\rangle = |00\rangle + \alpha |10\rangle + \beta |01\rangle + (\alpha^2/\sqrt{2})pq |20\rangle + (\alpha \beta)q |11\rangle + (\beta^2/\sqrt{2})qr |02\rangle , \]

The values of the coefficients are:
\[ \alpha = (\varepsilon/\kappa)(1 + 2C)^{-1} , \quad \beta = -\sqrt{Ng}(\gamma/2)^{-1}\alpha , \]
\[ p = 1 - 2C_1 , \quad q = (1 + 2C)/(1 + 2C - 2C_1) \]
\[ r = \sqrt{1 - 1/N} , \quad C_1 = C_1 (1 + \gamma/2\kappa)^{-1} \]

The probability density of two simultaneous transmission of photons is then:

\[ \left| \left\langle 00 | \hat{a}^2 |\psi\right\rangle \right|^2 = |\alpha^2 pq|^2 \]

This can be zero if \( p \) is zero.
\[ |\psi_c\rangle = \frac{\hat{a}|\psi\rangle}{\langle 00|\hat{a}|\psi\rangle} = |00\rangle + \beta q|01\rangle + \alpha pq|10\rangle, \]

\[ g^{(2)}(\tau) = |1 + \left(\frac{\Delta \alpha}{\alpha}\right) \exp\left[-\frac{(\kappa + \gamma/2)}{2} \tau\right] \]

\[ \times \left(\cos \Omega \tau + \frac{(\kappa + \gamma/2)}{2\Omega} \sin \Omega \tau\right) |^2 \]

\[ \left(\frac{\Delta \alpha}{\alpha}\right) = -2C_1 \left[\frac{2C}{\left(1 + 2C - 2C_1\right)}\right] \]
Regression of the field to steady state after the detection of a photon.
Classically $g^{(2)}(0) > g^{(2)}(\tau)$ and also $|g^{(2)}(0)-1| > |g^{(2)}(\tau)-1|$
Steady State:

Exchange of Excitation:
Cavity Mode and Atoms

\[ \Omega \]
The conditional field prepared by the click is:

$$A(t)|0\rangle + B(t)|1\rangle \text{ with } A(t) \approx 1 \text{ and } B(t) \ll 1$$

Mostly one prepares the vacuum!
Conditioned measurements in the language of correlation functions allow the study of the dynamics of the system.

Quantum conditioning, with photodetections, provides the most ideal times for controlling the evolution of the system.

Feedback.
Trigger the intensity-step with a fluctuation (photon) and measure the time evolution of the intensity as in $g^{(2)}(\tau)$. Can we feedback?
Conditional dynamics from the system wavefunction

\[ |\Psi_{ss}\rangle = |0, g\rangle + \lambda |1, g\rangle - \frac{2g}{\gamma} \lambda |0, e\rangle + \frac{\lambda^2 pq}{\sqrt{2}} |2, g\rangle - \frac{2g\lambda^2 q}{\gamma} |1, e\rangle \]

\[ \lambda = \langle \hat{a} \rangle, \quad p = p(g, \kappa, \gamma) \quad \text{and} \quad q = q(g, \kappa, \gamma) \]

A photodetection collapses the steady state into the following non-steady state from which the system evolves.

\[ \hat{a} |\Psi_{ss}\rangle \Rightarrow |\Psi_{collapse}\rangle = |0, g\rangle + \lambda pq |1, g\rangle - \frac{2g\lambda q}{\gamma} |0, e\rangle \]

\[ |\Psi(\tau)\rangle = |0, g\rangle + \lambda \left[ f_1(\tau) |1, g\rangle + f_2(\tau) |0, e\rangle \right] + O(\lambda^2) \]

Field \quad Atomic Polarization
Use passive feedback to stabilize the wavefunction

For times when the following relation holds, we see that the state resembles a steady state.

\[ f_2(T) = -\frac{2g}{\gamma} f_1(T) \]

Problem: The driving field at times, T, will not stabilize the state.

Solution: Change the driving field so that it will.

\[ \frac{\text{driving intensity } (\tau > T)}{\text{driving intensity } (\tau < T)} = f_1(T) \]
Plot to find times where quantum control will work.

\( (g, \kappa, \gamma)/2\pi = (30.0, 7.9, 6.0) \text{ MHz} \)
Theoretical prediction.
How long can we hold the system and then release it?
As long as we can!

How sensitive is it to detunings?
With our protocol we only operate well on resonance.

Where is the information stored?
New steady state.

What is quantum about this?
The detection of the first photon.

Deterministic source?
No, we mostly create the vacuum: $|0,g\rangle + \lambda |1,g\rangle + \ldots$