PLANAR SPIN SQUEEZING

Margaret Reid, Shi-Guo Peng,
Peter Drummond and Qiongyi He

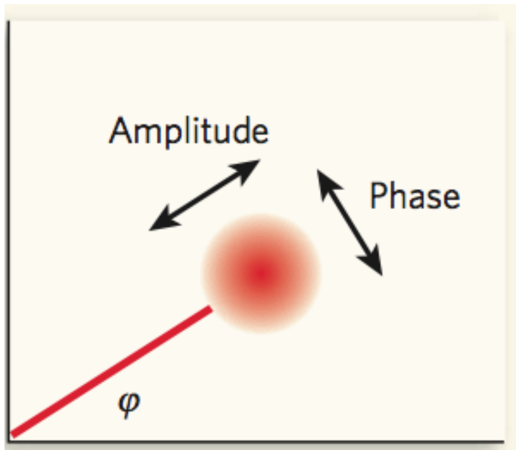
Centre for Atom Optics and Ultrafast Spectroscopy
Swinburne University of Technology,
Melbourne, Australia

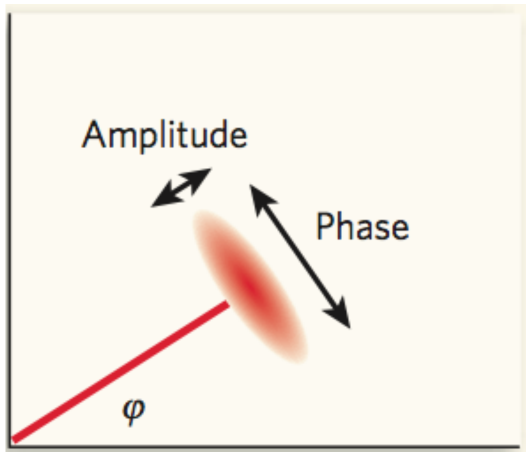
Outline

1. Squeezing with three observables
2. Planar Spin Squeezing: fundamental uncertainty in *two* spins
3. How to generate Planar Spin Squeezing in Ground State BEC
4. Applications

Squeezing

$$[X,Y]=2i \quad \Rightarrow \quad \Delta X\,\Delta Y \geq 1$$





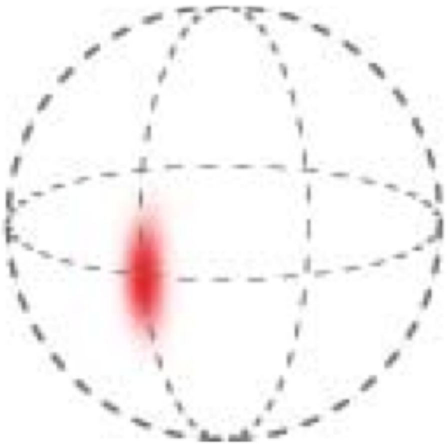
Pryde Nature

$$\Delta X = \Delta Y = 1 \qquad\qquad \Delta X < 1 \quad \Delta Y > 1$$

Spin Squeezing

$$\Delta J_X \, \Delta J_Y \geq \left| \left\langle J_Z \right\rangle \right| / 2$$

Kitagawa, Ueda
Wineland et al



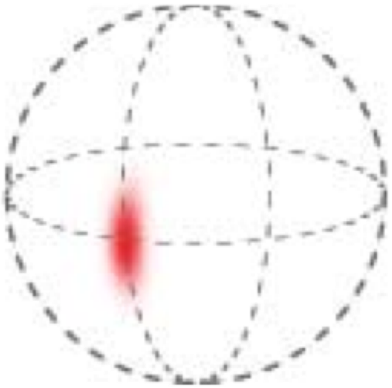
Gross et al, Nature

Fixed spin J

- Finite Hilbert space implies a *maximum* variance
- Three observables not two
- What if $\langle J_Z \rangle = 0$?
- More sophisticated uncertainty relations

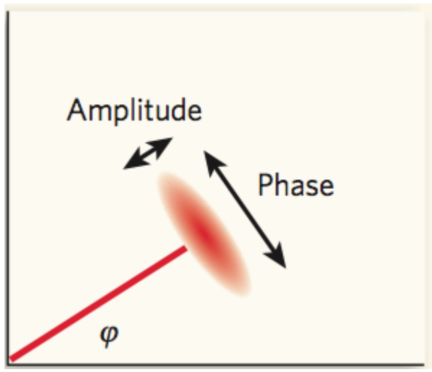
Lower Bound to Single Spin Squeezing

$$\Delta J_Y \Delta J_Z \geq \left|\left\langle J_X \right\rangle\right| / 2$$



$$Finite\ Size\ of\ J$$

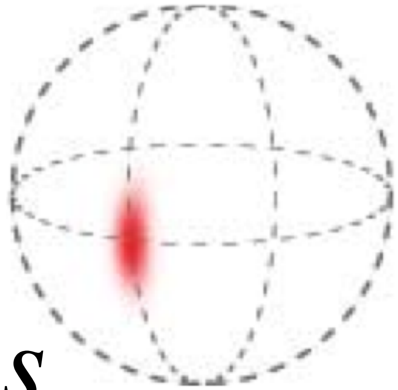
$$\Rightarrow \Delta J_Y \leq J$$

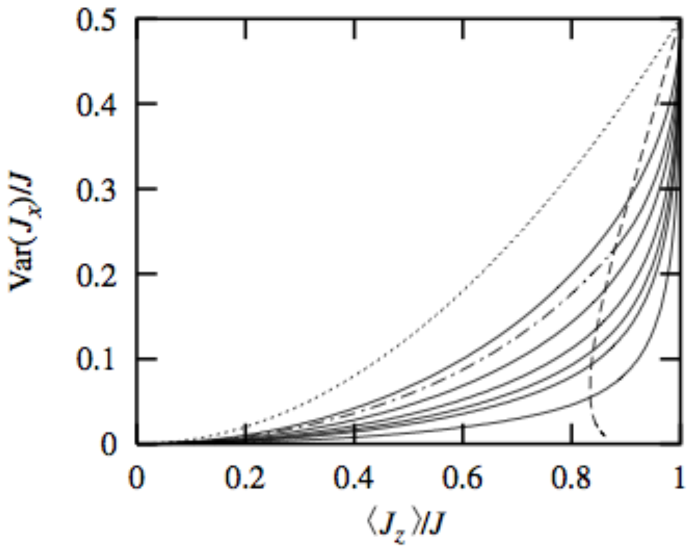


$$\Rightarrow \quad \Delta J_Z \geq \left|\left\langle J_X \right\rangle\right| / 2J$$

Extreme Squeezing: finding that precise lower bound for *perpendicular* spin

$$\Delta J_Y \Delta J_Z \geq \left|\left\langle J_X \right\rangle\right| / 2$$





More analysis

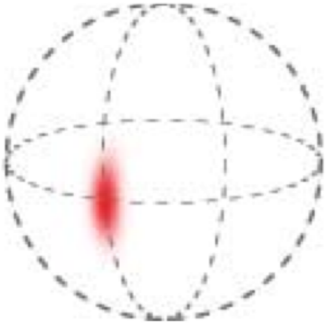
$$\Rightarrow \quad \Delta J_Z \geq F_J\left(\left|\left\langle J_X \right\rangle\right| / 2J\right)$$

Sorenson Molmer,PRL, 2000

$$\textit{Spin eigenstates are possible}: \parallel \textit{squeezing}$$

$$HUP \ \Rightarrow \quad \Delta J_Z = 0 \ \textit{only when} \ \left\langle J_X \right\rangle = \left\langle J_Y \right\rangle = 0$$

Uncertainty in *two* spins?
Planar quantum squeezing PQS



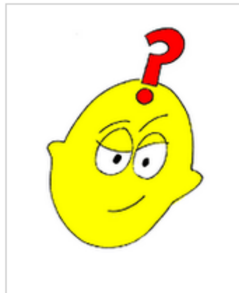
$$\Delta J_Y \Delta J_Z \geq \left|\langle J_X \rangle\right| / 2$$

$$(\Delta J_X)^2 + (\Delta J_Y)^2 + (\Delta J_Z)^2 \geq J \equiv N/2$$

Hofman, Takeuchi, PRA, 2003

$$\Delta J_Z = N/2 \quad \max$$

$$\Rightarrow \quad (\Delta J_X)^2 + (\Delta J_Y)^2 \geq 1/4 \ \ for\ J = 1/2\ ?$$



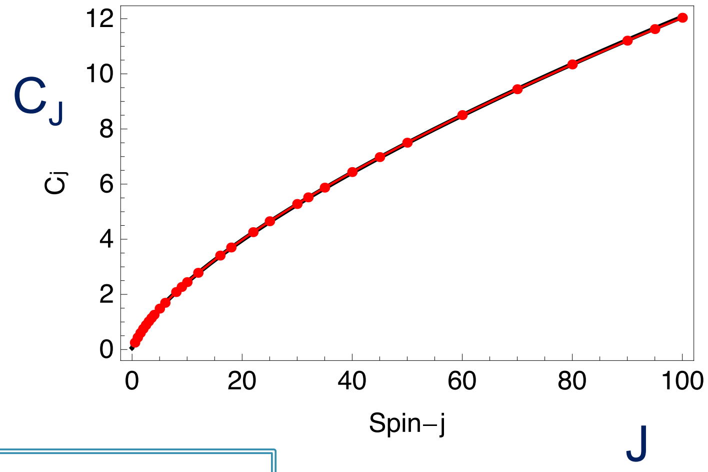
- Simultaneous spin eigenstate *not* possible
- We can't "squeeze" both perpendicular spins
  but parallel and perpendicular spins?
- How much?

Minimum Uncertainty Relation:

Solution for C_J

$$(\Delta J_X)^2 + (\Delta J_Y)^2 \geq C_J$$

$$|\Phi\rangle = norm \sum_{m=-J}^{m=J} R_m |J,m\rangle$$

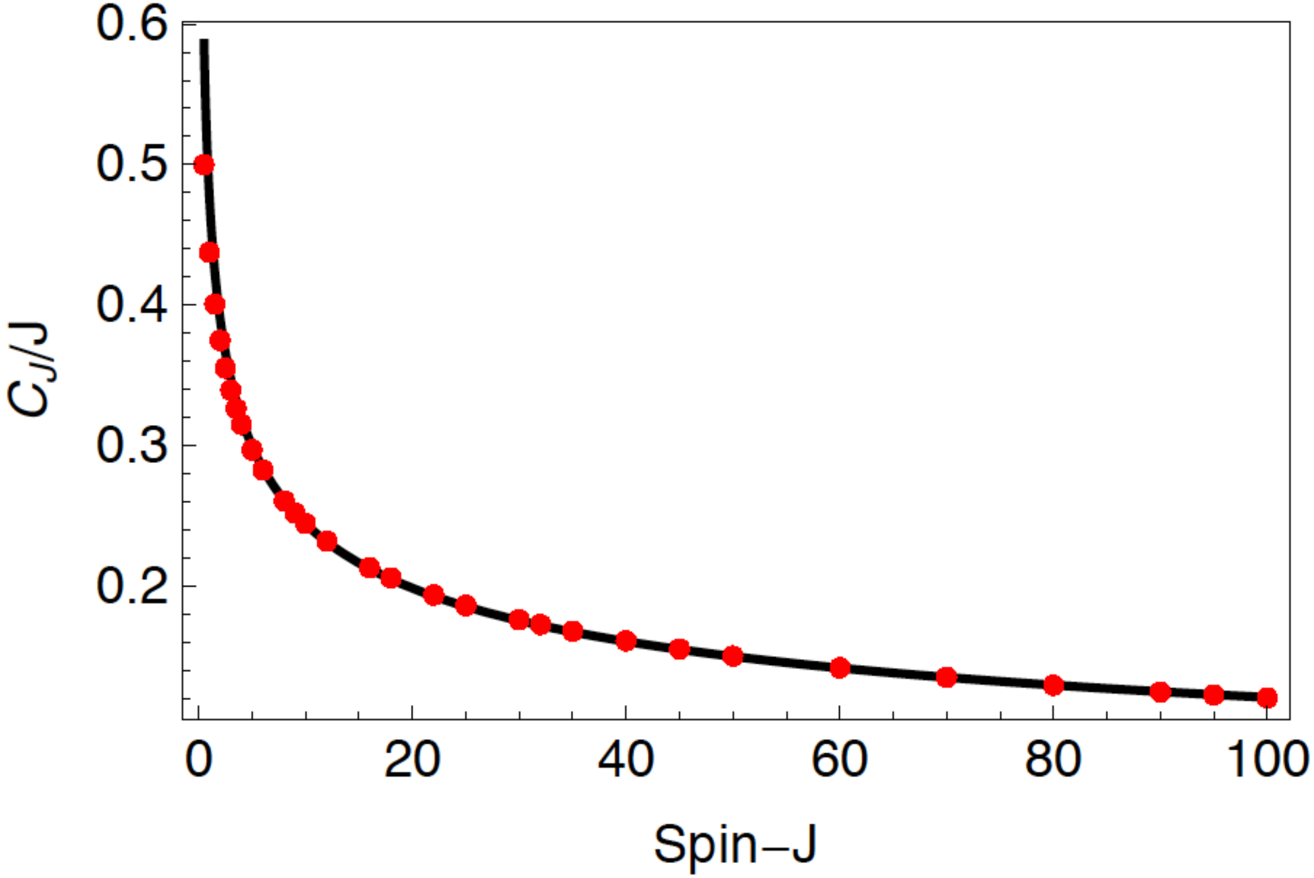


- Evaluate variances in terms of coefficients and minimise numerically
- Analytic asymptopic large J solutions- convert sums to integrals

$$\lim_{J \to \infty} C_J = 3(2J)^{2/3}/8$$

Solutions for C_J

$$(\Delta J_X)^2 + (\Delta J_Y)^2 \geq C_J$$

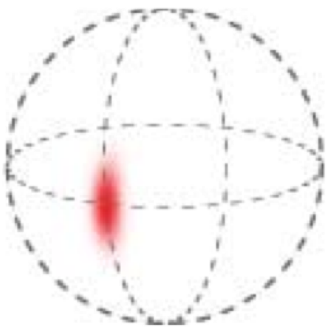


PQS Minimum Uncertainty States

$$(\Delta J_X)^2 + (\Delta J_Y)^2 = C_J$$

Large spin vector parallel to plane of squeezing
Maximum squeezing in direction of spin vector
Smaller variance reduction in plane of squeezing orthogonal to mean spin
Increased variance in third direction

$J \to \infty$



$$(\Delta J_X)^2 \sim (2J)^{2/3}/8$$

$$(\Delta J_Y)^2 \sim (2J)^{2/3}/4$$

$$(\Delta J_Z)^2 \sim (J^2/2)^{2/3}$$

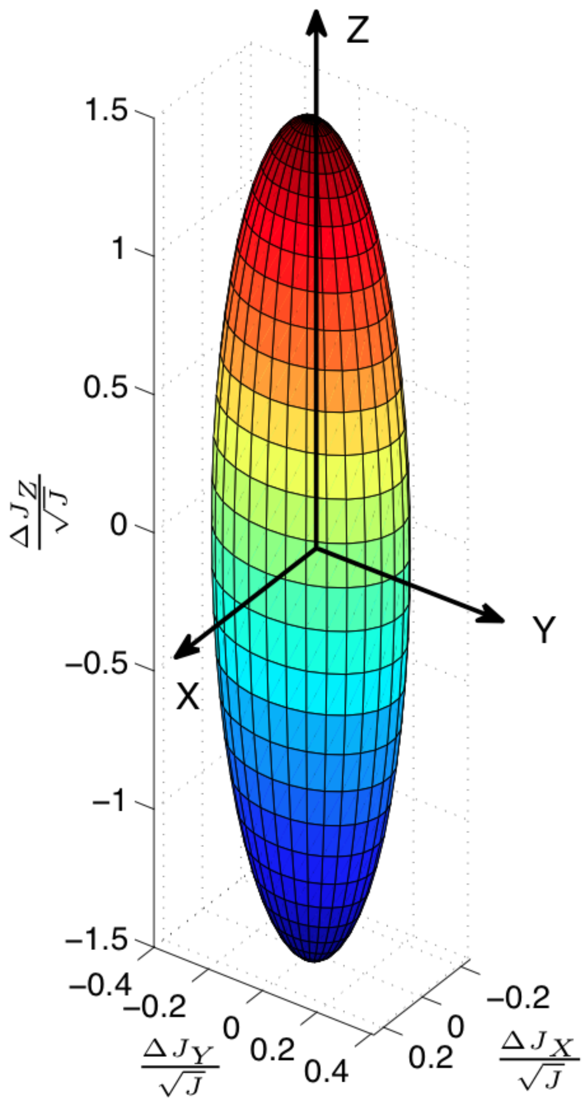
$$\Delta J_Y \Delta J_Z \sim \left|\langle J_X \rangle\right|/2$$

$$\langle J_X \rangle \sim J - \frac{1}{2}(J/4)^{1/3}$$

Angular momentum intelligent states as J becomes large

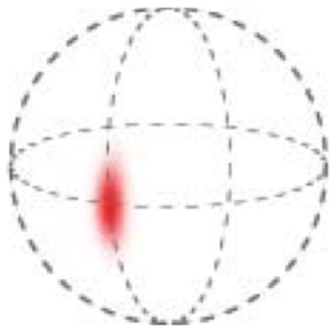
Aragone et al. J Phys A, 1974

Planar Squeezed Minimum Uncertainty States



$$(\Delta J_X)^2 + (\Delta J_Y)^2 = C_J$$

$$J \to \infty$$



J=50

$$(\Delta J_X)^2 \sim (2J)^{2/3}/8$$

$$(\Delta J_Y)^2 \sim (2J)^{2/3}/4$$

$$(\Delta J_Z)^2 \sim (J^2/2)^{2/3}$$

$$\langle J_X \rangle \sim J - \frac{1}{2}(J/4)^{1/3}$$

Planar spin squeezed state in X-Y plane

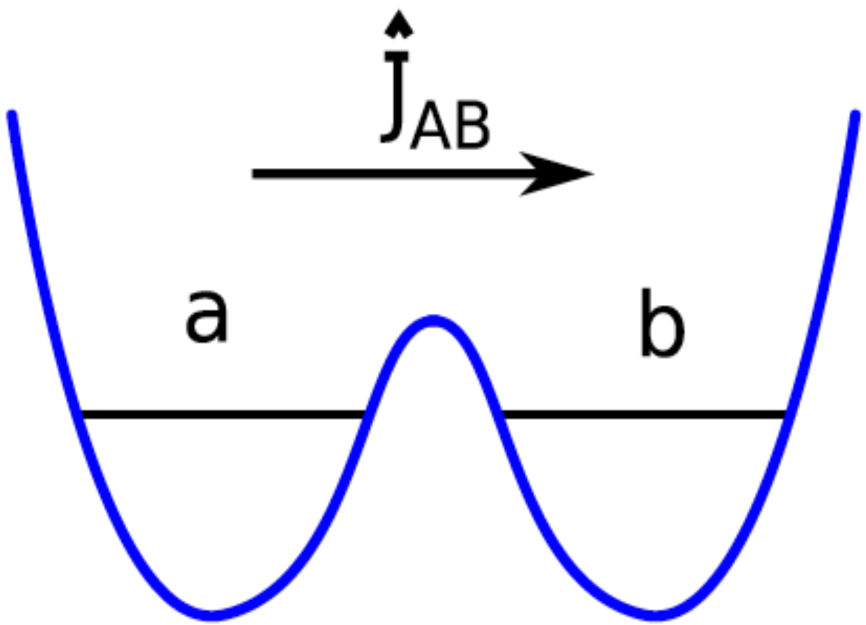
Angular momentum intelligent states in Y-Z plane

$$\Delta J_Y \Delta J_Z \sim |\langle J_X \rangle|/2$$

How can we generate the PQS states?

N atoms in two coupled wells BEC
(j=1/2, J=N/2)

$$H = \kappa(a^+ b + b^+ a) + \frac{g}{2}\left[a^+ a^+ aa + b^+ b^+ bb\right]$$



$$J_Z = (a^+ a - b^+ b)/2$$

$$J_X = (a^+ b + ab^+)/2$$

$$J_Y = (a^+ b - ab^+)/2i$$

$$N = a^+ a + b^+ b$$

What if the energy equals the variance sum?

$$H = \kappa(a^{+}b + b^{+}a) + \frac{g}{2}\left[a^{+}a^{+}aa + b^{+}b^{+}bb\right] = -g\left|J_{\parallel} - J_0\right|^2$$

$$J_{\parallel} = \{J_X, J_Y\}$$

$$J_{\perp} = J_Z$$

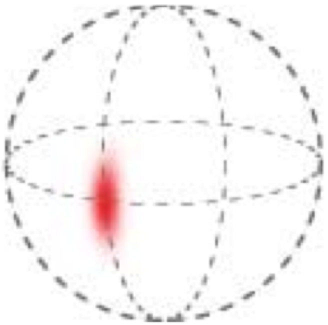
$$J_0 = (\kappa / g, 0)$$

ADJUST COUPLING SO THAT

$$J_0 = \langle J_X \rangle$$

$$H/(-g) = (\Delta J_X)^2 + (\Delta J_Y)^2 \geq C_J \; !$$

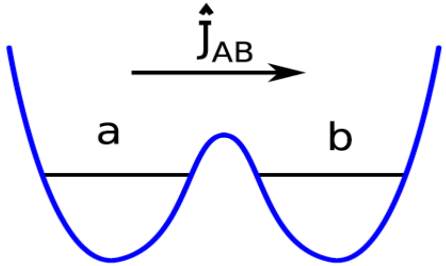
Indeed we solve: BEC two-well ground state

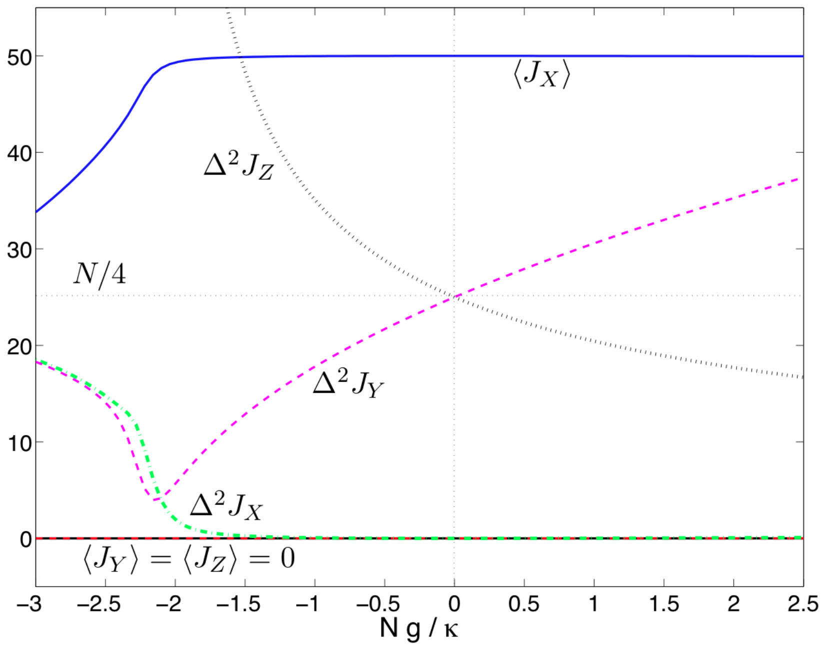


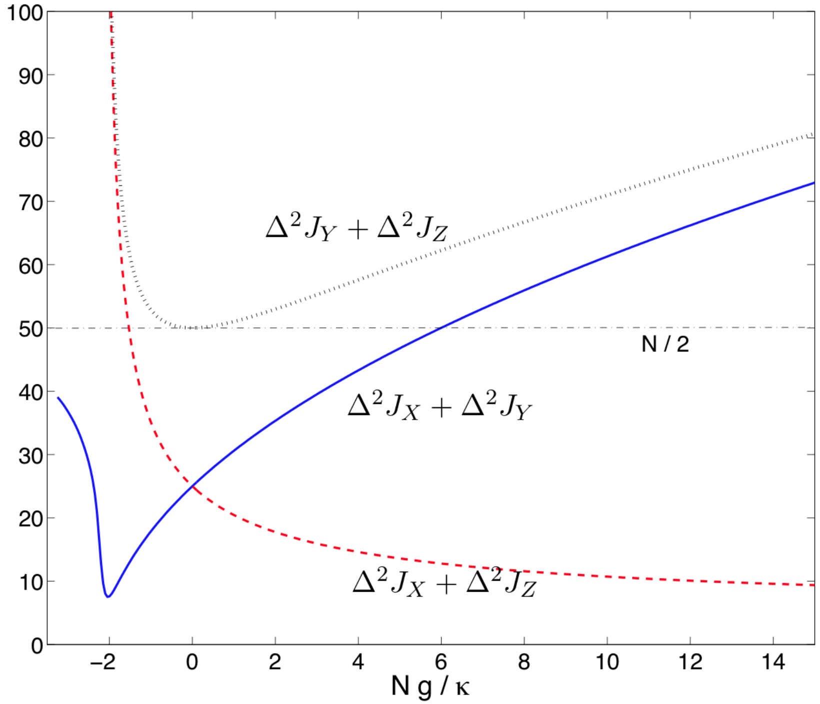
$$J = N/2$$

$$> (\Delta J_X)^2 + (\Delta J_Y)^2$$

$$\geq C_J$$

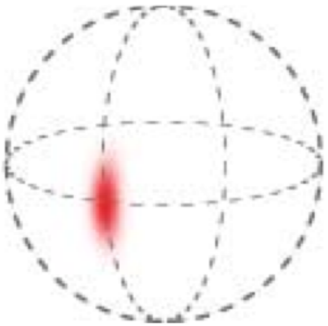




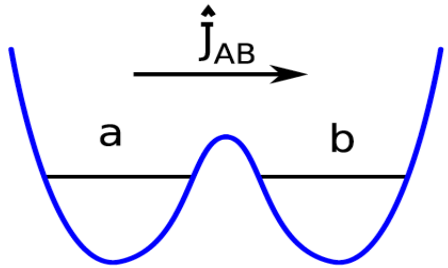


Attractive regime –g eg ^{7}Li
Repulsive regime +g eg ^{87}Rb

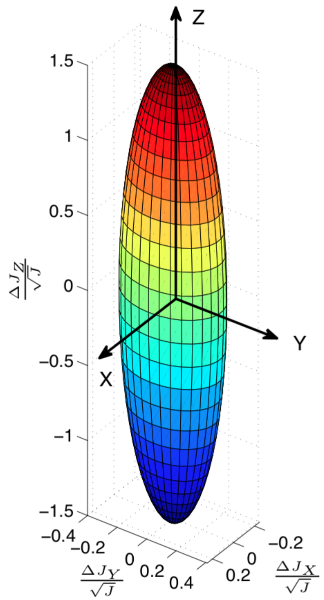
BEC two-well ground state

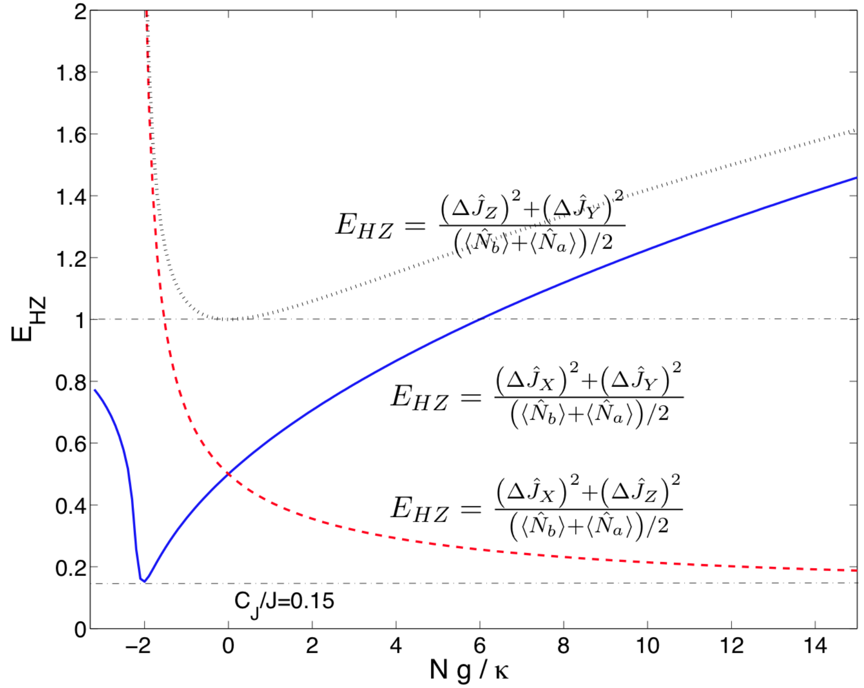


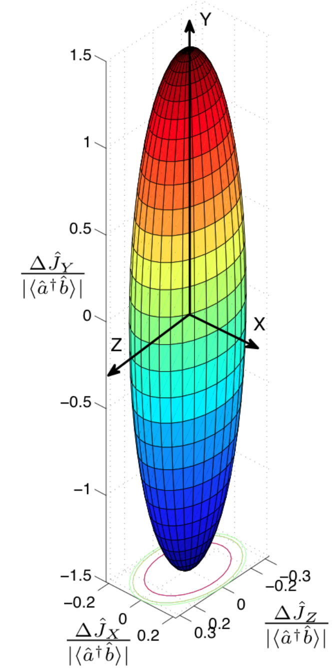
$$1 > \frac{(\Delta J_X)^2 + (\Delta J_Y)^2}{J} \geq C_J / J$$



N=100, J=50







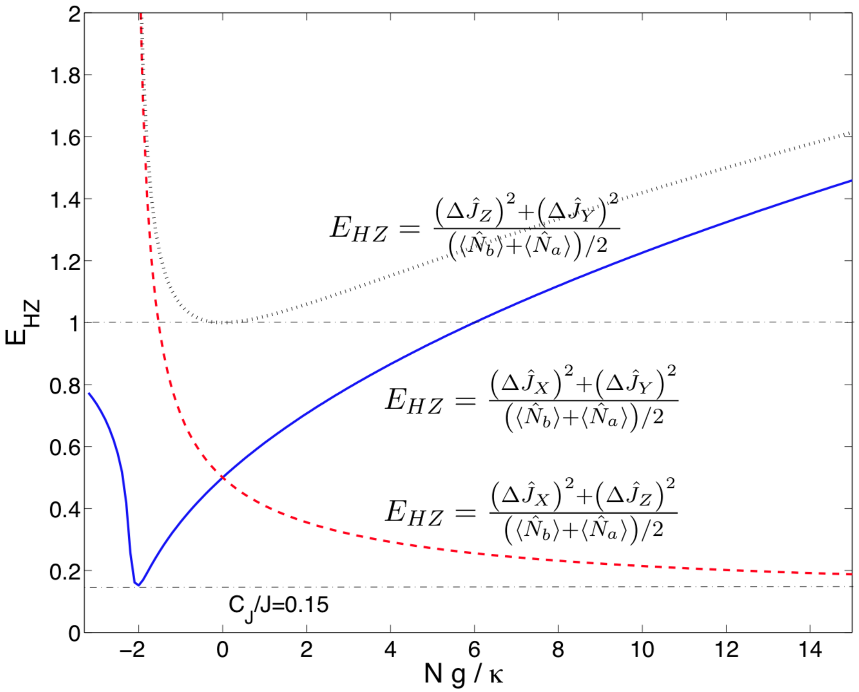
Maximum PQS in X-Y plane

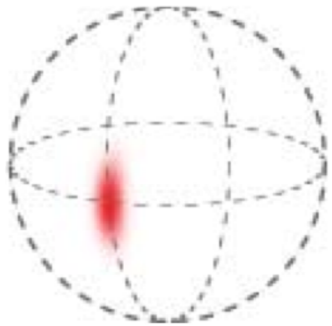
Near maximum PQS in X-Z plane

Two-mode Entanglement between Wells

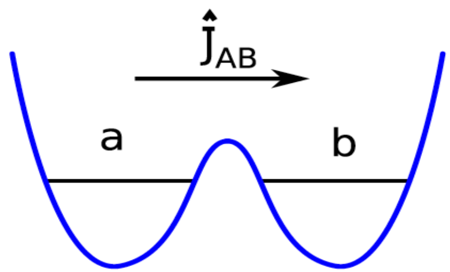
Hillery-Zubairy, PRL, 2006,
HZ entanglement criterion

$$E_{HZ} = \frac{(\Delta J_X)^2 + (\Delta J_Y)^2}{(N/2)} < 1$$



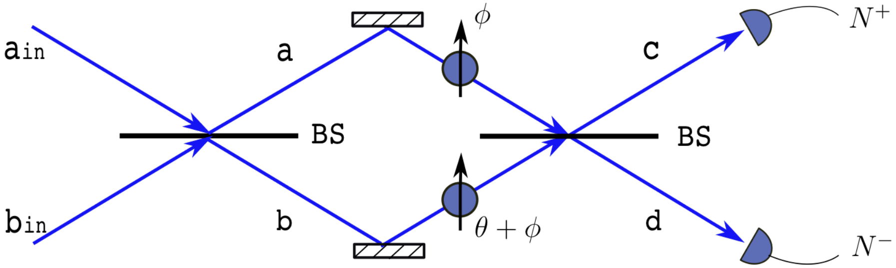


$$\Rightarrow \quad \rho \neq \sum_R P_R \rho_a \rho_b$$



Planar squeezing in XY plane implies
Entanglement between modes a and b

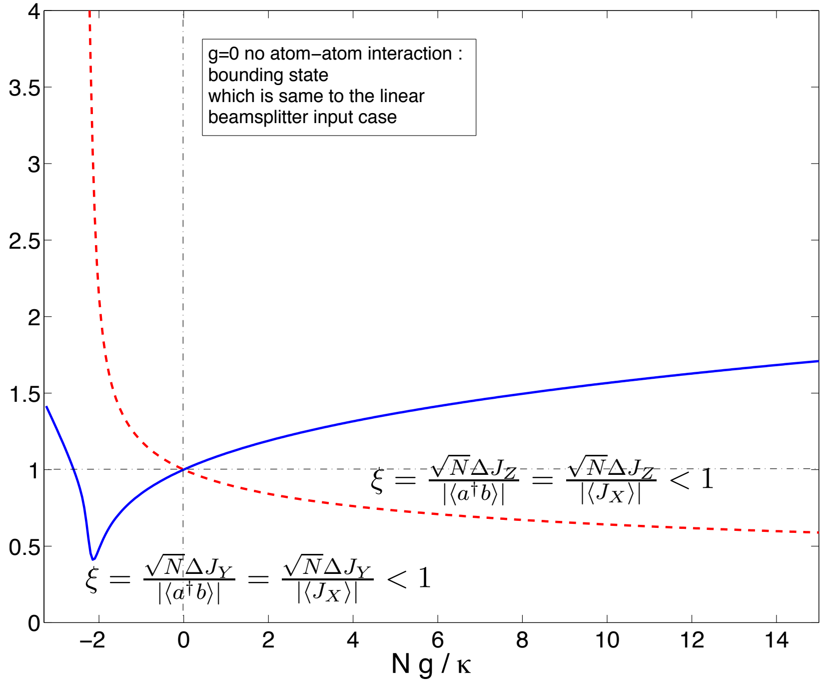
Particle Entanglement: Spin Squeezing parameter for enhanced interferometry



$$N^+ - N^- = J_X \cos\vartheta - J_Y \sin\vartheta$$

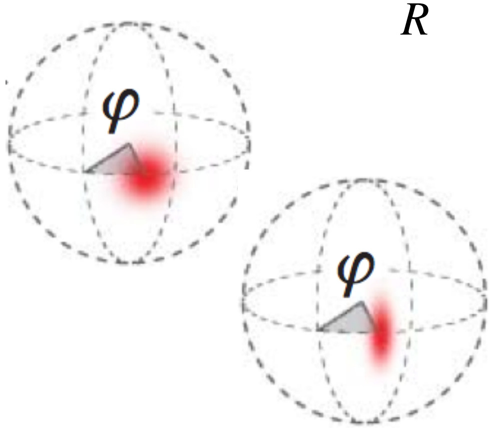
$$\xi^2 = \frac{N(\Delta J_\perp)^2}{(\langle J_\parallel \rangle^2)} < 1$$

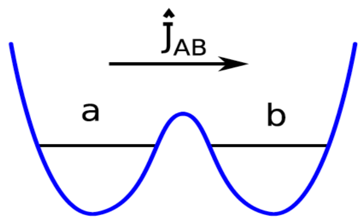
Wineland et al



$$\Rightarrow \quad \rho \neq \sum_R P_R \rho_1 \rho_2 \ldots \rho_N$$

Sorenson et al

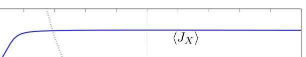




Gross et al, 2010

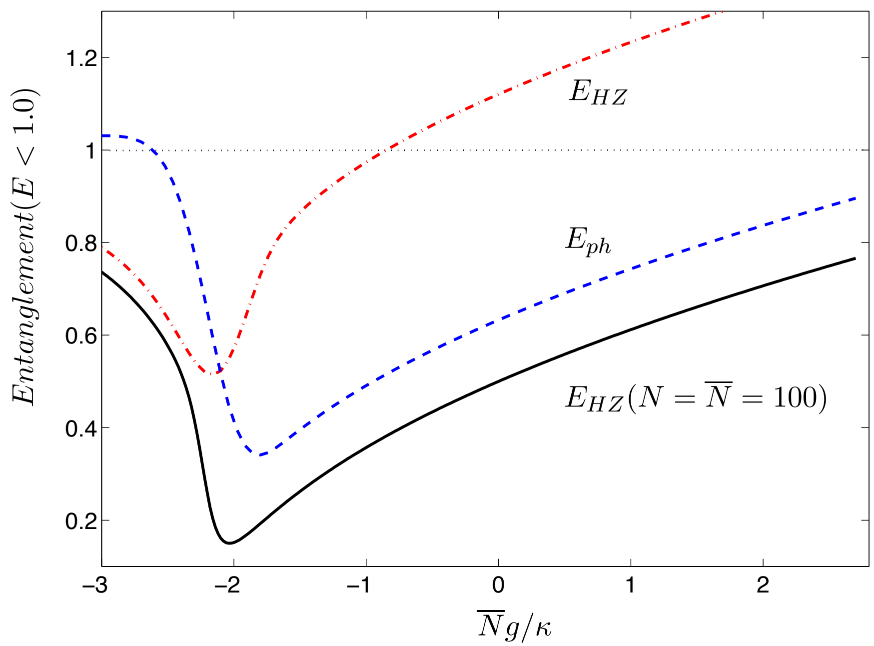
Entanglement between N particles

Lower limit of spin squeezing determined by the C_J value



Phase entanglement: reducing effect of number fluctuations

Measure HZ criterion but normalise number difference counts



$$\tilde{J}_Z = (a^+a - b^+b)/2N$$

$$\tilde{J}_X = (a^+b + ab^+)/2N$$

$$\tilde{J}_Y = (a^+b - ab^+)/2Ni$$

$$\hat{N} = a^+a + b^+b$$

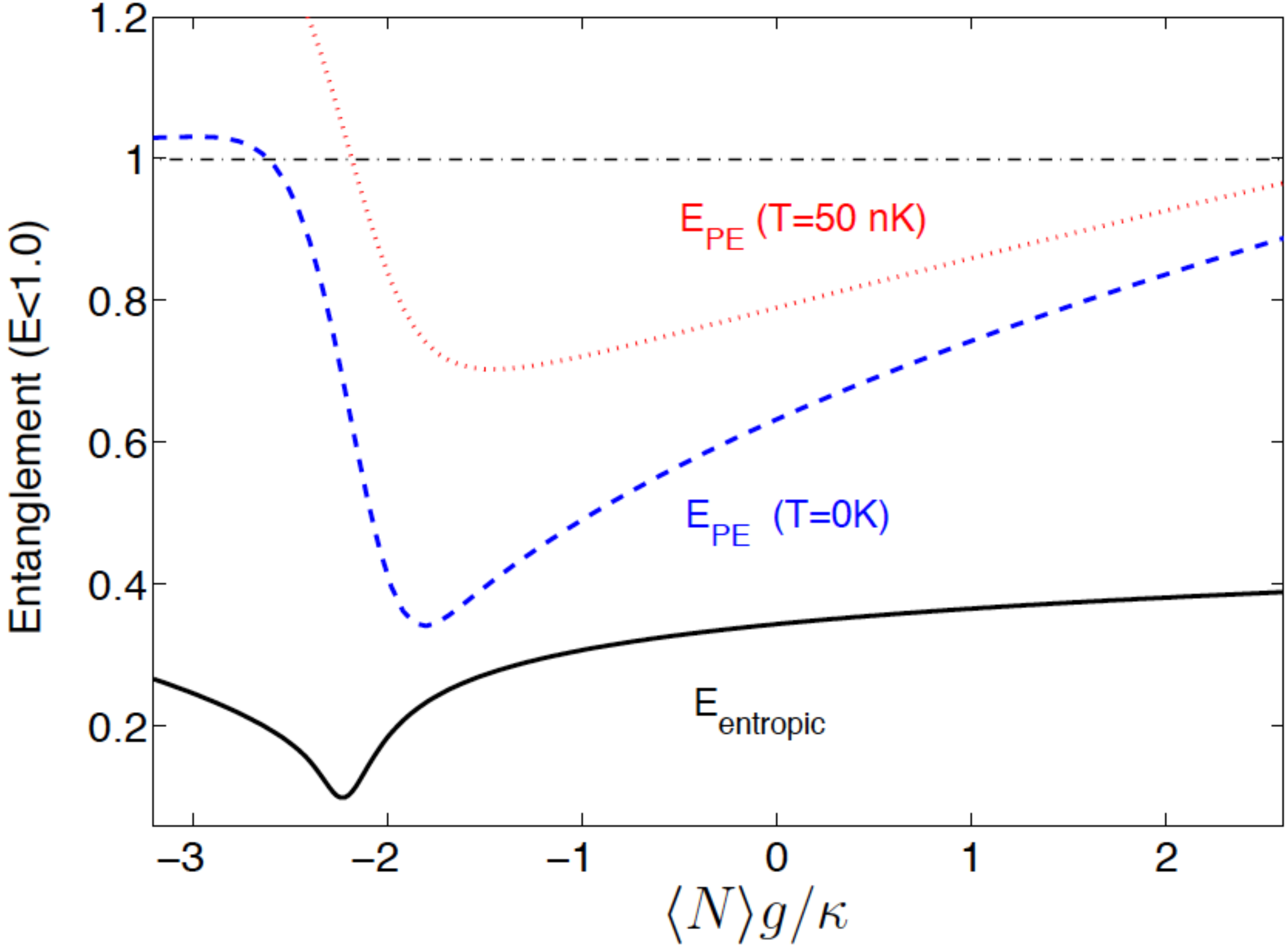
He et al, PRL 2011

N=100, Poisson number fluctuations

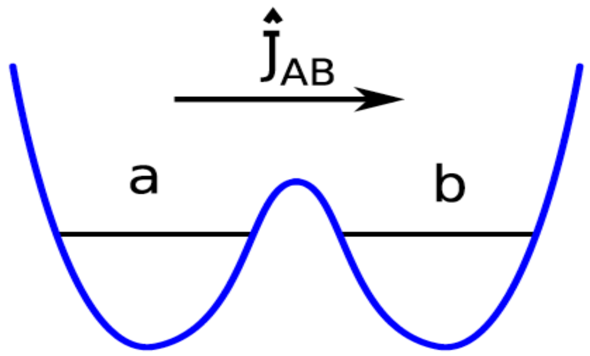
$$E_{ph} < 1 \quad \Rightarrow \quad \rho \neq \sum_R P_R \rho_a \rho_b$$

Entanglement between modes a and b

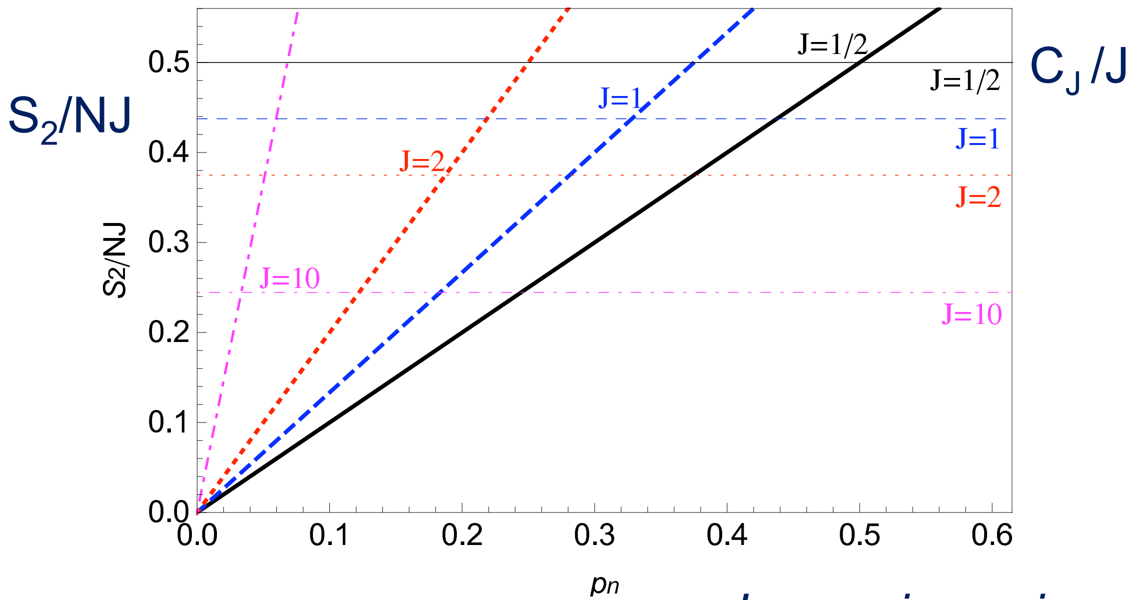
Finite T entanglement



Other Applications of Planar Squeezing?
Einstein-Podolsky-Rosen entanglement



N sites of spin J systems



Increasing noise

Is Cj criterion violated over many N sites?

Then spins must be entangled!

He et al, PRA, 2011

$$S_2 = (\Delta J_X^{total})^2 + (\Delta J_Y^{total})^2 < NC_J$$